Roll No. ______________

Total No. of Questions : 5]

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[2038]

B.E. I-Year Back Exam. April - 2008

MATHEMATICS - I

EB702Q

Time : 3 Hours

Maximum Marks : 60

Instructions to Candidates:

Attempt overall ***5*** *questions, selecting* ***one*** *question from each unit. All questions carry* ***equal*** *marks.*

Unit - 1

1. a) Light from a point source at A (7,0,0) strikes a small mirror at the origin O, the normal at which has direction ratios 2,1,2. Find the actual direction cosines of the reflected ray. (6)

b) i) Derive the formula of angle between two planes. (2)

ii) Find the shortest distance between the lines

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \quad \text{and} \quad \frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$$

Also find the coordinates of the points L and M where the line of shortest distance meets the lines. (4)

OR

1. a) A variable line in two adjacent positions has the direction cosines l, m, n and $l+\delta l,\ m+\delta m,\ n+\delta n$. Show that the small angle $\delta\theta$ between the two positions is given by $\delta\theta^2 = \delta l^2 + \delta m^2 + \delta n^2$. (3)

b) Find the equation of plane in the intercept form. (3)

c) Find the image of the line $\frac{x-1}{2}=\frac{y-2}{-1}=\frac{z-3}{2}$ in the plane $x-3y+z=9$. (3)

d) Find the equation to a sphere which passes through the point (α,β,γ) and the circle $x^2+y^2+z^2=a^2,\ z=0$. (3)

Unit - 2

2. a) Find the equation of the right circular cone generated by straight lines drawn from the origin to cut the circle through the three points (1, 2, 2), (2, 1, -2) and (2, -2, 1). (4)

b) Test for consistency the following system of equations and solve them if possible $5x+3y+7z=4;\quad 3x+26y+2z=9;\quad 7x+2y+10z=5.$ (4)

c) Find the eigen values and eigen vectors of the matrix $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$. (4)

OR

2. a) Find the equation of the right circular cylinder of radius 3 and axis $\frac{x-1}{2}=\frac{y-3}{2}=\frac{z-5}{-1}$. (4)

b) Define the rank of a matrix. Find the rank of matrix $A=\begin{bmatrix}1 & 3 & 5\\ 2 & 7 & 1\\ 3 & 2 & 4\end{bmatrix}$. (4)

c) State Cayley-Hamilton theorem. Verify it for the matrix A

$A=\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}$ find A^{-1}. (4)

Unit - 3

3. a) Find the asymptotes of the following curve $x^3+3x^2y-4y^3-x+y+3=0.$ (4)

b) Find the radius of curvature at $\left(\frac{3a}{2},\frac{3a}{2}\right)$, on the Folium of Descartes $x^3+y^3=3axy.$ (4)

c) Trace the curve $y^2(a+x)=x^2(a-x)$. (4)

OR

3. a) Find the cubic curve which has the same asymptotes as that of the curve $x^3-6x^2y+11xy^2-6y^3+x+y+1=0$ and which pass through the points (0, 0), (0, 1) and (1, 0). (4)

b) Show that the point of inflexion on the curve $y^2=(x-a)^2(x-b)$ lies on the line $3x+a=4b$. (4)

c) Trace the curve $r=a\,(1+\cos\theta)$. (4)

Unit - 4

4\. a) State and prove Euler theorem. (4)

b) A person being in a boat a k.m. from the nearest point of the beach wishes to reach as quickly as possible a point b km from that point along the shore. The ratio of his rate of walking to his rate of rowing is $\sec\alpha$. Prove that he should land at a distance $(b - a\cot\alpha)$ from the place to be reached. (4)

c) Find the area of the whole curve $a^2y^2 = a^2x^2 - x^4$. (4)

OR

4\. a) If $x^x y^y z^z = c$, show that at $x = y = z;\ \dfrac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$ (4)

b) Find the error in volume and total surface area of a right circular cone when its height is 4 cm and altitude 6 cm. When the errors in measurement of r and h be 1% in each. (4)

c) Show that the length of the curve $y = \log \sec x$ between the points where $x = 0$ and $x = \pi/3$ is $\log\left(2+\sqrt{3}\right)$. (4)

Unit - 5

5\. a) Find the surface and volume of the solid formed by revolving the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x - axis. (6)

b) Evaluate the double integral $\int_0^a \int_0^x \dfrac{x^3 dxdy}{\sqrt{x^2+y^2}}$. (6)

OR

5\. a) Find by double integration the area of the circle $r = a\sin\theta$ which lies outside the cardioid $r = a(1-\cos\theta)$. (6)

b) Change the order of integration in the following integral and evaluate

$$I = \int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dydx$$ (6)