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First B.E.

103 **Maths. II**

FIRST B.E. (Main/Back) EXAMINATION, 2006

(New Four-Year Scheme)

MATHEMATICS

Second Paper

Time allowed : Three hours

Maximum marks : 60

Attempt any five questions.

1. (a) Find by vector method the radial and transverse velocities and accelerations of a particle moving in a plane curve. 6

   (b) Find the values of constants a, b, c so that the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at P $(1, 2, -1)$ has a maximum magnitude 64 in a direction parallel to the Z-axis. 6

2. (a) A fluid motion is given by $q = (y + z)\, i + (z + x)\, j + (x + y)\, k$. Is this motion irrotational ? If yes find the

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velocity potential. Is the motion possible for an incompressible fluid ? 6

(b) Find the work done in moving a particle once round a square C formed by the lines $y = \pm 1$, $x = \pm 1$ in the xy-plane if the force is given by :

$$F = (x^2 + xy + z)\, i + (x^2 + y^2 - z)\, j + xyk$$

6

3. (a) A factory uses three different resources for the manufacture of two different products, 20 units of the resources A, 12 units of B and 16 units of C being available. 1 unit of first product requires 2, 2 and 4 units of the respective resources and 1 unit of second product requires 4, 2 and 0 units of the respective resources. It is known that the first product gives a profit of 2 monetary units per unit and the second 3. Formulate the linear programming problem. How many units of each product should be manufactured for maximizing the profit ? Solve it graphically. 6

(b) Solve the following L.P.P. by simplex method :

$$\text{Maximize } Z = -2x - y$$

$$\text{s.t.} \quad 3x + y = 3;$$

$$4x + 3y \geq 6;$$

$$x + 2y \leq 4$$

$$\text{and} \quad x, y \geq 0$$

6

4. (a) Define radial and transverse acceleration. A particle describes the curve $r = ae^{\theta}$ with constant angular velocity ω. Find the expression for radial and transverse velocities and accelerations. 6

(b) A point moves in a curve so that its tangential and normal accelerations are equal and the angular velocity of the tangent is constant. Show that the path is $s = Ae^{\psi} + B$. 6

5. (a) A particle P moves in a straight line OCP being attracted by a force $m\mu PC$, always directed towards C, while C moves along OC with a constant acceleration f. If initially C was at rest at the origin O and P was at a distance C from O moving with velocity V, find the distance x of P from O at any time t. 6

(b) A particle is moving under the acceleration $\frac{\mu}{x^2}$ towards the origin, where x is its distance from the origin which is also the centre of attraction. If it starts from rest at a distance a, show that the time taken from $x = \frac{3a}{4}$ to $x = \frac{a}{4}$ is $\frac{1}{3}$ times the time from $x = a$ to $x = 0$. 6

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6. (a) A torpedo moving in still water is subjected to a retardation K times the velocity. If u be its initial velocity, show that the velocity v and distance s after time t are given by $v = ue^{-kt}$:

$$s = \frac{u}{k}\left(1 - e^{-kt}\right)$$ 6

(b) If a particle is projected from the lowest point of a smooth vertical circle of radius r with velocity u, write the conditions that (i) particle oscillates on each side of the lowest point up to horizontal diameter,

(i) Particle leave the circle

(ii) Particle move right round the circle without leaving contact.

A train of mass M is standing on a railway curve of radius r which is banked up to suit a speed v. Show that there is a lateral thrust on the rails of magnitude

$$\frac{v^2 Mg}{\sqrt{v^4 + r^2 g^2}}.$$ 6

7. Define the order and degree of a differential equation.

Solve the following differential equations :

(i) $3x\left(1 - x^2\right)y^2 \frac{dy}{dx} + \left(2x^2 - 1\right)y^3 = ax^3$

(ii) $(x - y - 2)\,dx + (x - 2y - 3)\,dy = 0$

(iii) $(xy\sin xy + \cos xy)\,y\,dx + (xy\sin xy - \cos xy)\,x\,dy = 0$

2+4+3+3

8. Explain the term complimentary function and particular integral.

Solve the following differential equation :

(i) $\left(D^2 + 4\right)y = \tan 2x$

(ii) $\left(D^4 + D^2 + 1\right)y = e^{-x/2}\cos\left(\frac{\sqrt{3}}{2}x\right)$

(iii) $\left(D^2 + 2D + 1\right)y = x\cos x$

2+4+3+3