Roll No. ________________

Total No. of Questions : 5] [Total No. of Pages : 3

# [2038]

# B.E. I - Year Back Exam. April - 2008

# MATHEMATICS - II

# EB703Q

**Time : 3 Hours** **Maximum Marks : 60**

***Instructions to Candidates:***

*Attempt overall **5** questions, selecting **one** question from each unit. All questions carry **equal** marks.*

1. a) If $\vec{a}$ is a constant vector, show that $\nabla(\vec{a}.\vec{v})=(\vec{a}.\nabla)\vec{v}+\vec{a}\times(\nabla\times\vec{v})$ (6)

   b) Show that $\vec{F}=(2xy+z^3)\hat{i}+x^2\hat{j}+3z^2x\hat{k}$ is a conservative field. Find its scalar potential and also the work done in moving a particle from (1, -2, 1) to (3, 1, 4). (6)

**OR**

1. a) Find the values of the constant $n$ for which the vector $r^n\vec{r}$ is

   i) irrotational ii) solenoidal. $(\vec{r}=x\hat{i}+y\hat{j}+z\hat{k})$ (6)

   b) Find the work done in moving a particle once round a circle C in the $xy$-plane, if the circle has centre at the origin and radius 3 units and if the force field is given by $\vec{F}=(2x-y-z)\hat{i}+(x+y-z^2)\hat{j}+(3x-2y+4z)\hat{k}$. (6)

2. a) A firm manufacturing two types of electric items A and B can make a profit of Rs 20 per unit of A and Rs 30 per unit of B. Each unit of A requires 3 motors and 4 transformers while each unit of B requires 2 and 4 respectively. The supply of these per month is 210 and 300 respectively. Type B also requires a voltage stabilizer which has a supply restricted to 65 units per month. Formulate the L.P.P. for maximum profit and solve it graphically. (6)

   b) Solve the L.P.P.

   Minimize $z=x_1-3x_2+2x_3$

   s. t. $3x_1-x_2+2x_3\le 7$

   $-2x_1+4x_2\le 12$

   $-4x_1+3x_2+8x_3\le 10$

   and $x_1, x_2, x_3\ge 0.$ (6)

**OR**

2. a) Solve the following L.P.P. graphically : (6)

Minimize $z = 2x_1 + 3x_2$

s.t. $x_1 + x_2 \le 4$

$6x_1 + 2x_2 \ge 8$

$x_1 + 5x_2 \ge 4$

$x_1 \le 3$

$x_2 \le 3$

and $x_1, x_2 \ge 0$

b) Solve the L.P.P. (6)

Maximize $z = 3x_1 + 5x_2 + 4x_3$

s.t. $2x_1 + 3x_2 \le 8$

$2x_2 + 5x_3 \le 10$

$3x_1 + 2x_2 + 4x_3 \le 15$

and $x_1, x_2, x_3 \ge 0.$

3. a) A particle moves in a catenary $(s = c\tan\psi)$ and the direction of its acceleration at any point makes equal angles with the tangent and the normal to the path at the point. If the speed at the vertex $(\psi = 0)$ be $u$ show that the velocity and acceleration at any other point are given by $ue^{\psi}$ and $\frac{\sqrt{2}}{c}\, u^2\, e^{2\psi}\, \cos^2\psi.$ (6)

b) Show that the time of descent to the centre of force, the force varying inversely as the square of the distance from the centre, through the first half of its initial distance is to that through the last half as $(\pi+2):(\pi-2)$. (6)

**OR**

3. a) If the radial and transverse velocities of a point are always proportional to each other and this holds for accelerations also, prove that its velocity will vary as some power of the radius vector. (6)

b) A particle is performing a S.H.M. of period T about a centre O and it passes through a point P (OP = $b$) with velocity $v$ in the direction OP. Prove that the time which elapses before its return to P is $\frac{T}{\pi}\tan^{-1}\left(\frac{vT}{2\pi b}\right)$. (6)

4. a) Show that a particle projected upwards with a velocity U in a medium whose resistance varies as the square of the velocity, will return to the point of projection with a velocity $\frac{UV}{\sqrt{U^2+V^2}}$, where V is the terminal velocity. (6)

b) If the maximum and minimum speeds of a train on a track of radius $r$ are $u$ and v respectively, show that if the rail is banked up to an angle $\beta$ given by $\tan\beta = \frac{u^2+v^2}{2gr}$, then the lateral thrust of the fastest train is equal to the lateral thrust of the slowest one, assuming their weights to be equal. (6)

**OR**

4. a) A particle is projected in a medium whose resistance is proportional to the cube of the velocity and no other force acts on the particle. While the velocity diminishes from $v_1$ to $v_2$ and the particle traverses a distance $d$ in time $t$, show that $\frac{d}{t} = \frac{2v_1v_2}{v_1+v_2}$. (6)

b) A particle is hanging from a fixed point by a light cord one metre long and is started moving with an initial horizontal velocity such that the chord slackens when the particle is $\frac{5}{3}$ metres above the lowest point. Find the distance through which it will further rise. (6)

5. Solve the following differential equations.

i) $\frac{dy}{dx} = \frac{3y+2x+4}{4x+6y+5}$. (3)

ii) $(y\log x - 1)y\,dx = x\,dy$. (4)

iii) $\frac{d^4y}{dx^4} - a^4y = \cosh ax$. (5)

**OR**

5. Solve the following differential equations -

i) $(1+y^2)dx = (\tan^{-1}y - x)dy$. (3)

ii) $(1+xy)y\,dx + (1-xy)x\,dy = 0$. (4)

iii) $\frac{d^2y}{dx^2} + 4y = x\sin x$. (5)