3E1418

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B.E. II Year (Sem. III) (Old Scheme/Back) Examination, February/March - 2011

Advanced Engineering Mathematics

(Common to Mech., P & I, Auto. & IEM Engg.)

Time : 3 Hours] [Total Marks : 80

[Min. Passing Marks : 24

Attempt overall **five** *questions selecting one question from each unit. All questions carry equal marks.*

Use of following supporting material is permitted during examination.

(Mentioned in form No. 205)

1. Nil 2. Nil

UNIT-I

1 (a) Find the Fourier series of the function $f(x), -l < x < l$ defined as below

$$f(x)\begin{cases} \dfrac{l}{4}, & \text{when } -l < x < -\dfrac{l}{2} \\ \dfrac{x^2}{4}, & \text{when } -\dfrac{l}{2} < x < \dfrac{l}{2} \\ \dfrac{l}{4}, & \text{when } \dfrac{l}{2} < x < l \end{cases}$$

8

(b) Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$ hence prove that

$$\int_0^\infty \left[\frac{x\cos x - \sin x}{x^3}\right]\cos\frac{x}{2}\,dx = \frac{-3\pi}{16}$$

8

OR

1 (a) Using tabulated values of x and y given in the table, obtain a Fourier series up to third harmonic to represent the relation between x and y :

x:	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
y:	70	886	1293	1400	1307	814	–70	–886	–1293	–1400	–1307	–814

8

(b) Solve the following integral equation :

$$\int_0^\infty f(x)\sin sx\,dx = \begin{cases} 1, & 0 \le s < 1 \\ 2, & 1 \le s < 2 \\ 0, & s \ge 2 \end{cases}$$

8

UNIT-II

2 (a) Find the Laplace transform of $\sin\sqrt{t}$. Hence show that

$$L\left[\frac{\cos\sqrt{E}}{\sqrt{E}}\right] = \left(\frac{\pi}{s}\right)^{\frac{1}{2}} e^{-\left(\frac{1}{4s}\right)}.$$

5

(b) If $L\,f(t) = \bar{f}(s)$ then prove that $L\left[\frac{f(t)}{t}\right] = \int_s^\infty \bar{f}(u)\,du$ provided that $\underset{t\to\infty}{Lt}\ \frac{f(t)}{t}$ exists. Hence find $L\left(\frac{\sin at}{t}\right)$, does the $L\left(\frac{\cos at}{t}\right)$ exist.

5

(c) Find inverse Laplace transform of $\left(\frac{s^2}{s^4 + 4a^4}\right)$.

6

OR

2 (a) Solve $\frac{d^2y}{dt^2} + z\frac{dy}{dt} + y = t$

Given that $y(0) = -3,\ y(1) = -1$

8

(b) A string is stretched between the fixed points (0, 0) and (l,0) and released at rest from the initial deflection given by

$$f(x)=\begin{cases}\dfrac{2kx}{l}, & \text{when } 0<x<\dfrac{l}{2}\\[2mm] \dfrac{2k(l-x)}{l}, & \text{when } \dfrac{l}{2}<x<l\end{cases}$$

Find the deflection of the string at any time t. 8

UNIT-III

3 (a) Define an Analytic function and derive Cauchy-Reimann equation. Prove that the function $u=x^3-3xy^2+3x^2-3y^2+1$ satisfies Laplace equation and determine the corresponding analytic function $u+iv$. 8

(b) Expand $\dfrac{1}{z\left(z^2-3z+2\right)}$ in Laurent's series for the regions :

(i) $0<|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$ 8

OR

3 (a) Find the transformation which transform, the semi infinite strip bounded by $\vartheta=0, \vartheta=\pi$ and $u=0$ on to the upper half z plane. 8

(b) Evaluate $\displaystyle\int_{-\infty}^{\infty}\frac{x\sin\pi x}{x^2+2x+5}dx$. 8

UNIT-IV

4 (a) Solve $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0$ satisfying the conditions

$u(0,y)=u(l,y)=u(x,0)=0$ and $u(x,a)=\sin\left(\dfrac{n\pi x}{l}\right)$. 8

(b) Prove that

$\dfrac{d}{dx}\left[x^n J_n(x)\right]=x^n J_{n-1}(x)$ and $\dfrac{d}{dx}\left[J_n(x)\right]=\dfrac{n}{x}J_n(x)-J_{n+1}(x)$

Hence show that $x^n J_n(x)$ is the solution of

$$x\frac{d^2y}{dx^2}+(1-2n)\frac{dy}{dx}+xy=0.$$

OR

4 (a) State and prove Rodrigue's formula. 4

(b) Prove that :

(i) $(2n+1)xP_n = (n+1)P_{n+1} + nP_{n-1}$ (ii) $nP_n = xP_n' - P_{n-1}'$. 4

(c) Find the series solution of :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + \left(x^2 - n^2\right) y = 0$$ 8

UNIT-V

5 (a) The following table gives the value of a function at equal intervals :

x	0.0	0.5	1.0	1.5	2.0
$f(x)$	0.3989	0.3521	0.2420	0.1295	0.0540

Evaluate $f(1.8), f'(1.5), \int_0^1 f(x)dx$, stating the formula used. 8

(b) Use Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule to evaluate the following $\int_0^1 \frac{dx}{1+x^2}$. Hence obtain the approximate value of π in each case. 8

OR

5 (a) Evaluate $\Delta^6 (ax-1)\left(bx^2-1\right)\left(cx^3-1\right)$. 5

(b) Find the value of $f(5)$ from the following table by using Lagrange's interpolation formula. 5

(c) Prove that :

$$\delta\left[f(x)g(x)\right] = \mu f(x)\ \delta g(x) + \mu g(x)\ \delta f(x)$$ 6