B

16 50

II B. E.

3101

Maths.-III

II B. E. (III Semester) (Main/Back) EXAMINATION, 2006

(New Four-Year Semester Scheme)

(Computer Engineering Common with I.T.)

MATHEMATICS-III

First Paper

Time allowed : Three hours

Maximum marks : 80

Attempt any five questions. All questions carry equal marks.

1. (a) Solve :

$$\left(x^2 D^2 + 4xD + 2\right) y = e^x$$

6 (2)

(b) Solve :

$$\left(1 - x^2\right)\frac{d^2 y}{dx^2} + x\frac{dy}{dx} - y = x\left(1 - x^2\right)^{3/2}$$

5

PTO

(c) Solve :

$$\cos x \frac{d^2y}{dx^2} + \sin x \frac{dy}{dx} - 2\cos^3 x . y = 2\cos^5 x$$

5

2\. (a) Solve by the method of variation of parameters :

$$\frac{d^2y}{dx^2} + 4y = 4\tan 2x$$

8

(b) Solve in series :

$$\left(1-x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$$

8 4

3\. (a) Solve the following partial differential equations :

(i) $9\left(p^2 z + q^2\right) = 4$

(ii) $(x+y)(p+q)^2 + (x-y)(p-q)^2 = 1$

4+6

(b) Find the complete integral by Charpit's method :

$$p = (qy+z)^2$$

6

4\. (a) A tightly stretched flexible string has ends at $x = 0$ and $x = l$. At $t = 0$, the string is given a shape defined

by $y(x) = \mu x(l-x)$, μ being a constant and then released. Find the displacement of the string for any x and $t > 0$.

8

(b) Prove that :

$$L\left\{\frac{\sin^2 x}{x}\right\} = \frac{1}{4}\log\left(\frac{P^2+4}{P^2}\right)$$

Hence deduce that :

$$\int_0^\infty \frac{\sin^2 x}{x^2}\,dx = \frac{\pi}{2}$$

8

5\. (a) Find the inverse Laplace transform with the help of convolution theorem :

$$L^{-1}\left\{\frac{P}{\left(P^2+a^2\right)\left(P^2+b^2\right)}\right\}$$

5

(b) Solve :

$$\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2}$$

subject to $u(0,t) = 0$, $u(2,t) = 0$

and $u(x,0) = 20\sin 2\pi x$; $u_t(x,0) = 0$

6

(c) Find the Fourier cosine transform of e^{-x^2}. 5

6. (a) Solve :

$$\frac{\partial V}{\partial t}=\frac{\partial^2 V}{\partial x^2};\ x>0,\ t>0$$

subject to the conditions :

(i) $V=0$ when $x=0,\ t>0$

(ii) $V=\begin{cases}1, & 0<x<1 \text{ when } t=0\\ 0, & x\geq 1\end{cases}$

(iii) $V(x,t)$ bounded

8

(b) Find $f(1.5), f(7.5), f'(7)$ from the following table :

x	$f(x)$	y
1	1	1^3
2	8	2^3
3	27	3^3
4	64	4^3
5	125	5^3
6	216	6^3
7	343	7^3
8	512	8^3

8

7. (a) Find the first and second derivative at 1.1 for the data :

x	$f(x)$
1	0
1.2	0.1280
1.4	0.5440
1.6	1.2960
1.8	2.4320
2.0	4.0000

8

(b) Use Runge-Kutta fourth order method to solve :

$$\frac{dy}{dx}=-2xy^2\ ,\quad y(0)=1$$

with $h=0.2$ for $x=0.2$ and $x=0.4$

8

8. (a) Solve :

$$y_{n+2}-2\cos\alpha\, y_{n+1}+y_n=\cos\alpha n$$ 5

(b) By using Newton Raphson's method, find the root of $x^4-x-10=0$ which is nearer to $x=2.0$, correct to three places of decimal. 5

(c) Find the inverse of the following matrix with the help of Cayley-Hamilton theorem : 6

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

10
12
12
4
38