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II B.E. (III Semester)

Math. III

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II B.E. (Third Semester) (Main/Back)

EXAMINATION, Dec., 2005

(New Four Year Semester Scheme)

(Branch : Civil Engineering)

MATHEMATICS—III

Time : Three Hours

Maximum Marks : 80

Attempt any *Five* questions.

All questions carry equal marks.

1. Solve the following differential equations :

(a) $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=\frac{1}{(1-x)^2}$. 8

(b) $\left(\frac{d^2y}{dx^2}+y\right)\cot x+2\left(\frac{dy}{dx}+y\tan x\right)=\sec x$. 8

2. (a) Use the method of variation of parameters to solve :

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x^2e^x.$$ 8

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(b) Solve in series :

$$x(1-x)\frac{d^2y}{dx^2}+(1-5x)\frac{dy}{dx}-4y=0. \qquad 8$$

3. (a) Find the general integral of :

$$p\cos(x+y)+q\sin(x+y)=z. \qquad 8$$

(b) Find the complete integral by Charpit's method :

$$2xz-px^2-2qxy+pq=0. \qquad 8$$

4. (a) An insulated rod of length l has its ends A and B kept at 0°C and 100°C respectively until steady state condition prevail. If the temperature of B is then suddenly reduced to 0°C and kept so, while that of end A is maintained, find the temperature $u(x, t)$ at distance x from A at time t. 8

(b) Find the Laplace transforms of :

$$t^2e^t\sin 4t. \qquad 8$$

5. (a) If :

$$f[f(t)]=\bar{f}(s),$$

then prove that :

$$f[tf(t)]=-\frac{d}{ds}(\bar{f}(s)).$$

Use above theorem to find the inverse Laplace transform of :

$$\log\sqrt{\left(1+\frac{9}{s^2}\right)}. \qquad 4+4$$

(b) Use Laplace transform theory to solve :

$$(D^2+1)x=t\cos 2t,\ x(0)=x'(0)=0. \qquad 8$$

6. (a) Use Laplace transform theory to solve the following partial differential equation :

$$\frac{\partial u}{\partial t}=2\frac{\partial^2 u}{\partial x^2}$$

Subject to the conditions,

$u(0,t)=0,\ u(5,t)=0$ and $u(x,0)=10\sin 4\pi x$. 8

(b) Define unit step function and find its Laplace transform. Find the inverse L.T. of :

$$\frac{s}{s^2-w^2}.e^{-as}. \qquad 2+3+3$$

7. (a) Find the Fourier series to represent $f(x)=x-x^2$ in the interval $-1<x<1$. 8

(b) Obtain the first three cosine terms and the constant term in the Fourier series for y, where :

x	y
0	4
1	8
2	15
3	7
4	6
5	2

8

8. (*a*) Show that the following function is harmonic and determine its conjugate function :

$$u = e^x(x\cos y - y\sin y).$$ 4+4

(*b*) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. Hence find the image of $|z| < 1$. 5+3