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1491

B. E. - II Year (Sem. III) Examination, December – 2007

Mathematics - III

(Common to EC, Applied EC, EIC and Bio Medical)

Time : 3 Hours]

[Total Marks : 80

[Min. Passing Marks : 24

*Attempt overall **5** questions selecting **one** question from each unit. All questions carry **equal** marks.*

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. Nil 2. Nil

SECTION - A

1 (a) Find the Laplace transform of $\sin\sqrt{t}$. Hence show that $L\left(\frac{\cos\sqrt{t}}{\sqrt{t}}\right)=\left(\frac{\pi}{s}\right)^{\frac{1}{2}} e^{-\frac{1}{4s}}$.

(b) Use Laplace transformation to solve

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=t$$

Given $y(0)=-3,\ y(1)=-1$.

OR

1 (a) Find the inverse Laplace transform with the help of convolution theorem $\frac{s^2}{s^4 - a^4}$.

(b) Solve $\frac{\partial^2 u}{\partial t^2} = 9 \frac{\partial^2 u}{\partial x^2}$

B.C. $u(0, t) = 0$, $u(2, t) = 0$

$u(x, 0) = 20 \sin 2\pi x - 10 \sin 5\pi x$ and

$u_t(x_0) = 0$

SECTION - B

2 (a) Find the Z transform of $u_n = c^n \cos an$, $n \geq 0$ where a is real.

(b) Find the Fourier series of the function $f(x)$, $-l < x < l$ defined as below

$$f(x) = \begin{cases} \frac{l}{4}, & when \ -l < x < -\frac{l}{2} \\ \frac{x^2}{4}, & when \ -\frac{l}{2} < x < \frac{l}{2} \\ \frac{l}{4}, & when \ \frac{l}{2} < x < l \end{cases}$$

OR

2 (a) Find the inverse Z transform of

$$F(z) = \frac{1}{(z-3)(z-2)}$$

If R.O.C. in (i) $|z| < 2$, (ii) $2 < |z| < 3$, (iii) $|z| > 3$

(b) In a machine, the displacement y of a given point is given for a certain angle θ as follows

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
y	0.9	8.0	7.2	5.6	3.6	1.7	0.5	0.2	0.9	2.5	4.7	6.8

SECTION - C

3 (a) Find the Fourier sine and cosine transform of the following function

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$$

(b) $\frac{\partial \theta}{\partial t} = k \frac{\partial^2 \theta}{\partial x^2}$, $x > 0$, $t > 0$

with the boundary conditions : $\theta = \theta_0$ when $x = 0,\ t > 0$ and the initial conditions : $\theta = 0$ when $t = 0,\ x > 0$.

OR

3 (a) Find $F(x)$. If its sine transform is $\frac{s}{1+s^2}$.

(b) Use the method of Fourier integral to determine the displacement $y(x,\ t)$ of an infinite string, given that the string is initially at rest and that the initial displacement is

$$f(x),\ -\infty < x < \infty$$

Show that the solution can also be put in the form

$$y(x,\ t) = \frac{1}{2}[f(x-ct) + f(x+ct)]$$

SECTION - D

4 (a) Define Analytic function, derive Cauchy-Riemann equation for them and show that continuity does not imply differentiability by considering the function $|z|^2$.

(b) State and prove Cauchy's theorem.

OR

4 (a) Show that the polar form of Cauchy-Riemann equations are $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \ \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$. Derive these.

(b) Find the transformation which transforms the semi-infinite strip bounded by $v = 0, \ v = \pi$ and $u = 0$ on to the upper half of z plane.

SECTION - E

5 (a) Expand $\frac{1}{z(z^2 - 3z + 2)}$ in Laurent series for the region (a) $0 < |z| < 1$ (b) $1 < |z| < 2$ (c) $|z| < 2$.

(b) Prove that

$$\int_0^{2\pi} \frac{\cos^2 3\theta \, d\theta}{1 - 2p\cos\theta + p^2} = \pi \frac{(1 - p + p^2)}{1 - p}, \ (o < p < 1).$$

OR

5 (a) Evaluate $\int_C \frac{z^2 e^{zt}}{z^2 + 1} dz$ where C is the circle $|z| = 2$ and t is a quantity independent of z.

(b) Show that

$$\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta} = \int_0^{2\pi} \frac{d\theta}{a + b\sin\theta} = \frac{2\pi}{\sqrt{a^2 + b^2}}, \ a > b > 0.$$