3E1491

Roll No. : ______________ Total Printed Pages : 4

**3E1491**

**B.Tech. (Sem.III) (Main/Back) Examination, January - 2009**
**(3AI1) Mathematics - III (App.Elect. & I.Eng.)**
**(3EI1) Mathematics - III (Electro. Ins. & Cont. Eng.)**
**(3EC1) Mathematics - III (Electro. & Comm. Eng.)**
**(3BM1) Mathematics - III (Bio-Med. Eng.)**

Time : **3** Hours] [Total Marks : **80**
[Min. Passing Marks : **24**

*Attempt five questions in all selecting one question from each Unit.*
*Schematic diagrams must be shown wherever necessary.*
*Any data you feel missing may suitably be assumed and stated clearly.*
*Units of quantities used / calculated must be stated clearly.*

Use of following supporting material is permitted during examination.
*(Mentioned in form No. 205)*

1. Nil 2. Nil

## UNIT - I

1 (a) (i) Find $L\left[\sin\sqrt{t}\right]$ where $L$ denotes the Laplace transform.

(ii) Find $L^{-1}\left[\log\left(\frac{s+1}{s-1}\right)\right]$, where $L^{-1}$ denotes the inverse Laplace transform.

4+4

(b) Apply convolution theorem to evaluate :

$$L^{-1}\left[\frac{1}{s^2(s^2-a^2)}\right]$$

8

2 (a) Solve by Laplace transformation method :

$$\left(D^2+9\right)y=\cos 2t\text{, given that } y(0)=1,\ y\left(\frac{\pi}{2}\right)=-1$$

8

(b) Use Laplace transformation theory to solve $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$, $u(x,0)=3\sin 2\pi x$, $u(0,t)=0$, $u(1,t)=0$ where $0<x<1, t>0$.

8

## UNIT - II

3 (a) Find the Fourier series for $f(x) = x + x^2$, $-\pi < x < \pi$. Hence show that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$ 8

(b) The following values of $y$ gives the displacement of a certain machine part for the rotation $x$ of the flywheel :

| $x$ | $0$ | $\frac{\pi}{3}$ | $\frac{2\pi}{3}$ | $\pi$ | $\frac{4\pi}{3}$ | $\frac{5\pi}{3}$ | $2\pi$ |
|---|---|---|---|---|---|---|---|
| $y$ | 1.98 | 2.15 | 2.77 | –0.22 | –0.31 | 1.43 | 1.98 |

Express y in a Fourier series upto the third harmonic. 8

4 (a) Find the Z-transform of $u_n = c^n \cos an, n \geq 0$ where $a$ is real. 8

(b) Find the inverse Z-transform of $\frac{1}{(z-a)^2}$

(i) $|z| < a$ (ii) $|z| > a$ 4+4

## UNIT - III

5 (a) Obtain the Fourier transform of

$$f(x) = x^2, \text{ for } |x| \leq a$$
$$= 0, \text{ for } |x| > a$$

8

(b) Find the Fourier sine and cosine transform of $f(x) = e^{-x}, x \geq 0$.

Also, show that $\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}, m > 0$. 8

6 (a) Prove the given theorem :

If $\bar{F}(s)$ and $\bar{G}(s)$ are the Fourier transforms of $f(x)$ and $g(x)$ respectively, and if $h(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(u) g(x-u)\, du$

then $\bar{F}[h(x); s] = \bar{F}(s) \cdot \bar{G}(s) = \bar{F}[f(x); s]\, \bar{F}[g(x); s]$. 8

(b) Using Fourier transform, show that the solution of the partial differential equation :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, -\infty < x < \infty, t \geq 0$$

Satisfying the conditions :

(i) $u = f(x)$ when $t = 0$

(ii) $\frac{\partial u}{\partial t} = 0$ when $t = 0$

(iii) $u(x, t), \frac{\partial u}{\partial x}$ both tend to zero as $x \to \pm\infty$ can be written in the form

$$u(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \bar{F}(s) \cos(cst) e^{-isx} ds.$$

Where $\bar{F}(s)$ is the Fourier transform of $f(x)$. 8

## UNIT - IV

7 (a) If $f(z) = \begin{cases} \frac{x^2 y (y - ix)}{x^6 + y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$

Prove that $\left[\frac{f(z) - f(0)}{z - 0}\right] \to 0$ as $z \to 0$ along any radius vector but not as $z \to 0$ along the curve $y = ax^3$. Is this function differentiable at $z = 0$ ? 8

(b) Prove that the relation $W = \frac{iz+2}{4z+i}$ transforms the real axis in the $Z$-plane into a circle in the $W$-plane. Find the centre and radius of the circle and the point in the $Z$-plane which is mapped on the centre of the circle. 8

8 (a) State and prove Cauchy's integral theorem. 8

(b) Evaluate the following integral by using Cauchy's integral formula :

$$\frac{1}{2\pi i}\int_C \frac{e^{tz}}{z^2+1}dz,\ t>0,\ \text{where } C \text{ is the circle } |z| = 3.$$

8

## UNIT - V

9 (a) Expand $\frac{1}{z\left(z^2-3z+2\right)}$ in Laurent's series for the regions :

(i) $0<|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$

8

(b) Find the residues of $f(z)=\frac{z^2-2z}{(z+1)^2\left(z^2+4\right)}$ at all its poles in the finite plane.

8

10 (a) Evaluate the integral : $\int_C \frac{z^2}{(z-1)^2(z-2)}dz,\ c:|z|=2.5$

8

(b) Evaluate the following integral by contour integration :

$$\int_0^\infty \frac{dx}{\left(x^2+1\right)^2}.$$

8