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3E1486

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B. Tech. (Sem. III) (Main/Back) Examination, February - 2010
(Common for 3EE6.1 & 3EX1)
(Mathematics)

Time : 3 Hours] [Total Marks : 80
[Min. Passing Marks : 24

*Attempt overall **five** questions in all. Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitable be assumed and stated clearly.*

Use of following supporting material is permitted during examination.
(Mentioned in form No. 205)

1. Nil 2. Nil

1 (a) Find the Laplace transform of the following functions

(i) $e^{-2t}\cos^2 t$ (ii) $t^2 e^t \sin t$ 8

(b) Using Laplace transform solve $\frac{d^2x}{dt^2}+\frac{dx}{dt}=2$. Given that $x=3$ at $t=0$ and $\frac{dx}{dt}=1$ at $t=0$. 8

OR

1 (a) Find the Laplace transform of $\sin\sqrt{x}$. Hence show that

$$L\left\{\frac{\cos\sqrt{x}}{\sqrt{x}}\right\}=\left(\frac{\pi}{s}\right)^{1/2} e^{-1/4s}.$$

8

(b) Apply convolution theorem to evaluate $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$. 8

2 (a) Find the Fourier sine transform of $f(x)=e^{-x},\ x\geq 0$. Also show that $\int_0^\infty \frac{x\sin mx}{x^2+1}dx=\frac{\pi}{2}e^{-m},\ m>0$. 8

(b) Solve the differential equation by using Fourier sine transform

$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2},\ x>0,\ t>0$ subject to the condition

(a) $u(0,t)=0$

(b) $u(x,0)=\begin{cases}1, & 0<x<1\\ 0, & x\geq 1\end{cases}$ *when* $t=0$ and

(c) $u(x,t)$ is bounded. 8

OR

2 (a) Find the Fourier sine and cosine transforms of

$$f(x)=\begin{cases}x, & 0<x<1\\ 2-x, & 1<x<2\\ 0, & x\geq 2\end{cases}$$

8

(b) Solve the heat equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2},\ -\infty<x<\infty\ \ t>0$, subject to $u(x,0)=f(x)$ where $f(x)=\begin{cases}u_0, & |x|<a\\ 0, & |x|>a\end{cases}$. 8

3 (a) Expand $f(x) = |\cos x|$ in a Fourier series in the interval $(-\pi, \pi)$. 8

(b) The turning moment T units of the crank shaft of a steam engine is given for a series of values of the crank angle θ in degrees :

$\theta°$	0	30	60	90	120	150	180
T	0	5224	8097	7850	5499	2626	0

Find the first four terms in a series of sine of represent T. Also calculate T when $\theta = 75°$. 8

OR

3 (a) Find the Fourier series to represent the function $f(x) = x^2$, $-\pi < x < \pi$ and hence deduce that $\frac{\pi^2}{8} = 1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \ldots\ldots$. 8

(b) Find the equation of the curves for which the functional $\int_0^1 \{(y')^2 + 12\,xy\}\,dx$, $y' = \frac{dy}{dx}$ with $y(0) = 0$ and $y(1) = 1$ can be extremised. 8

4 (a) Show that the function $u + iv = f(z)$, where

$$f(z) = \begin{cases} \dfrac{x^3(1+1') - y^3(1-1')}{x^2 + y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

Satisfy the Cauchy-Riemann equations at the origin, yet $f'(0)$ does not exist. 8

(b) Evaluate the following integral by using Cauchy's integral formula $\oint_C \frac{e^{2z}}{(z+1)^4}\,dz$ where C is the circle $|z| = 2$. 8

OR

4 (a) If $f(z)=u+iv$ be an analytic function of z and

$$u+v=\frac{2\sin 2x}{e^{2y}+e^{-2y}-2\cos 2x} \quad \text{find } f(z).$$

8

(b) Show that the transformation $W=z+\frac{a^2-b^2}{4z}$ transforms, the circle of radius $\frac{1}{2}(a+b)$, centre at the origin in the z-plane into ellipse of semi-axes a, b in the W-plane.

8

5 (a) State and prove Taylor's series.

8

(b) Evaluate the following integral by Contour integration : $\int_0^\infty \frac{dx}{1+x^4}$.

8

OR

5 (a) Expand $\frac{1}{z(z^2-3z+2)}$ in Laurent series for the region :

(i) $0<|z|<1$

(ii) $1<|z|<2$

(iii) $|z|<2$

8

(b) Evaluate $\oint_C \frac{z\cos z}{\left(z-\frac{\pi}{2}\right)^2}dz$ where C is the circle $|z-1|=1$.

8