3E1486

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B.Tech. IInd Year IIIrd Semester(Main/Back) Examination, Feb. - 2011

Common for 3 EE 6.1 and 3EX1

Mathematics - III

Time : 3 Hours **Maximum Marks : 80**

Min. Passing Marks : 24

Instructions to Candidates:

*Attempt overall **five** questions selecting **one** question from **each** unit. All questions carry **equal** marks.*

Unit - I

1. a) Find Laplace Transform of : $t^2.e^t.\ Sin\ 4t$ (8)

b) Use Laplace transform theory to solve :

$(D^2+1)x = t\,Cos\,2t,\ x(0) = x'(0) = 0$. (8)

OR

a) Apply the convolution theorem to btain

$$L^{-1}\left[\frac{s^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}\right].$$ (8)

b) Solve the partial differential equation by using Laplace transform method :

$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2},\ x>0,\ t>0$ subject to the conditions

$y\,(0,t) = 10\ Sin\ 2t,\ y\,(x,0) = 0,\ y_t\,(x,0) = 0,$

and $\lim\limits_{x\to\infty} y(x,t) = 0$. (8)

Unit - II

2. a) Solve the integral equation : (8)

$\int_0^\infty f(x)\,Cos\,\lambda x\,dx = e^{-\lambda}$.

b) Use Fourier Transform theory to solve : (8)

$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, given that $u_x(0,t) = 0$ and $u(x,0) = \begin{cases} x\ , & 0 \le x \le 1 \\ 0\ , & x > 1 \end{cases}$

$u\ (x,\ t)$ is bounded and $x > 0,\ t > 0$.

OR

a) Find *f(x)* if its Fourier sine transform is $\frac{1}{s}.e^{-as}$. Hence deduce $\bar{F}_s^{-1}\left(\frac{1}{s}\right)$. **(8)**

b) Using Fourier sine transform solve the partial differential equation

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, x>0, t>0.$$

Subject to the conditions

$$u(0,t)=0$$

$$u(x,0)=\begin{cases}1 \text{ when } & 0<x<1\\ 0 \text{ when } & x\geq 1\end{cases}$$

It may be assumed that *u(x,t)* is bounded; also u and $\frac{\partial u}{\partial x}$ approach zero as $x\to\infty$. **(8)**

Unit - III

3. a) Find the Fourier series to represent $f(x) = x - x^2$ in the interval $-1 < x < 1$. **(8)**

b) The following values of y give the displacement of a certain machine part for the rotation x of the flywheel,

x :	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
y :	1.98	2.15	2.77	-0.22	-0.31	1.43

Express y in a Fourier series upto the third harmonic. **(8)**

OR

a) Use the idea of calculus of variations to prove that the shortest distance between two given points in a plane is a straight line. **(8)**

b) Finel the extremals of the isoperimetric problem

$$V[y(x)]=\int_0^1\left(x^2+y'^2\right)dx \text{ given } \int_0^1 y^2\,dx=2, y(0)=0, y(1)=0.$$ **(8)**

Unit -IV

4. a) Show that the function $u+iv=f(z)$, where

$$f(z)=\begin{cases}\frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, & z\neq 0\\ 0 & , z=0\end{cases}$$

Satisfies the Cauchy - Riemann equations at the origin, yet $f'(0)$ does not exist. **(8)**

b) Evaluate $\int_{1-i}^{2+i}(2x+iy+1)\,dz$ along the two paths.

i) $x=t+1,\ y=2t^2-1$;

ii) the straight line joining $1-i$ and $2+i$. **(4+4)**

OR

a) Show that under the transformation $W=\frac{z-i}{z+i}$, real axis in the z-plane is mapped into the circle $|W|=1$. What portion of the z-plane corresponds to the interior of the circle? **(8)**

b) Evaluate the following integrals : **(4+4)**

i) $\int_C \frac{1-2z}{z(z-1)(z-2)}\,dz$, where C is the circle $|z|=1.5$.

ii) $\int_C \frac{e^{2z}}{(z+1)^4}\,dz$, where C is the circle $|z|=2$.

Unit - V

5. a) Expand the following function in Laurent's series. $\frac{1}{z(z-1)(z-2)}$ for

i) $|z-1|<1$ ii) $|z|>2$. **(4+4)**

b) Show that

$$\int_0^{2\pi}\frac{d\theta}{(5-3\,Sin\,\theta)^2}=\frac{5\pi}{32}$$ **(8)**

OR

a) Evaluate

$$\int_C \frac{(12z-7)}{(z-1)^2\,(2z+3)}\,dz$$

where C is the circle i) $|z|=2$, ii) $|z+i|=\sqrt{3}$. **(8)**

b) Show that

$$\int_0^{\infty}\frac{Sin\,mx}{x}\,dx=\frac{\pi}{2}$$ **(8)**