3E1486

Roll No. : ____________

Total Printed Pages : 4

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B. Tech. (Sem.III) (Main/Back) Examination, January - 2009
(3EE6) Mathematics - III
(Electrical Engineering)

Time : **3** Hours] [Total Marks : **80**

[Min. Passing Marks : **24**

Attempt five questions in all selecting one question from each Unit.
Schematic diagrams must be shown wherever necessary.
Any data you feel missing may suitably be assumed and stated clearly.
Units of quantities used / calculated must be stated clearly.

Use of following supporting material is permitted during examination.
(Mentioned in form No. 205)

1. Nil 2. Nil

UNIT - I

1 (a) If $L\{f(t)\} = \bar{f}(s)$, then prove that $L\left\{\frac{1}{t}f(t)\right\} = \int_s^\infty \bar{f}(s)\, ds$ provided that the integral exists. Hence obtain $L\left\{\frac{1}{t}(\cos at - \cos bt)\right\}$. 8

(b) Find the bounded solution $y(x, t)$, $0 < x < 1$, $t > 0$ of the boundary value problem

$$\frac{\partial y}{\partial x} - \frac{\partial y}{\partial t} = 1 - e^t, \quad y(x, 0) = x.$$

8

OR

1 (a) Apply the convolution theorem to obtain $L^{-1}\left\{\frac{s}{(s^2 + a^2)^2}\right\}$. 8

(b) Use Laplace transform theory to solve that equation :

$$(D^2 + 9)y = \cos 2t, \quad y(0) = 1, \quad y\left(\frac{\pi}{2}\right) = -1.$$

8

UNIT - II

2 (a) Find the Fourier transform of

$$f(x) = 1 - x^2, \quad |x| < 1$$
$$= 0, \quad |x| > 1$$

Hence evaluate $\int_0^\infty \frac{x \cos x - \sin x}{x^3} \cos \frac{x}{2}\, dx$. 8

(b) Using Fourier cosine transform, solve

$$\frac{\partial \theta}{\partial t} = c^2 \frac{\partial^2 \theta}{\partial x^2}$$

Subject to the conditions :

(i) $\theta = 0$ when $t = 0$, $x \geq 0$;

(ii) $\frac{\partial \theta}{\partial x} = -\mu$, a constant, when $x = 0$ and $t > 0$.

Assume that $\theta(x, t)$ and $\frac{\partial \theta}{\partial x}$ both tend to zero as $x \to \infty$. 8

OR

2 (a) Apply the fast Fourier-transform algorithm, find the discrete Fourier transform of the sequence $\{1, 0, -1\}$. 8

(b) Obtain the Fourier transform of

$$f(x) = x^2, \; for \; |x| \leq a$$
$$= 0, \; for \; |x| > a$$

Hence evaluate

$$\int_0^\infty \cos\left(\frac{as}{2}\right)\left\{(a^2 s^2 - 2) \sin as + 2as \cos as\right\} / s^3 \, ds.$$

8

UNIT - III

3 (a) Find the Fourier series to represent the following function : $f(x) = x \sin x$, $-\pi \leq x \leq \pi$, Hence. deduce that

$$\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1.3} - \frac{1}{3.5} + ...$$

8

(b) Obtain the expansion for y from the following table up to first harmonic :

x	0	1	2	3	4	5
y	9	18	24	28	26	20

8

OR

3 (a) Find the Fourier series for $f(x) = x + x^2$, $-\pi < x < \pi$. Hence show that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots$. 8

(b) Find the curve, passing through the two points (x_1, y_1), (x_2, y_2) which when rotated about the $x-axis$ gives minimum surface area. 8

UNIT - IV

4 (a) Show that under the transformation $w = \frac{z-i}{z+i}$, real axis in the $z-$plane is mapped into the circle $|w| = 1$. What portions of the $z-$plane corresponds to the interior of the circle ? 8

(b) If $f(z_0) = \int_C \frac{3z^2 + 7z + 1}{z - z_0} dz$, where C is the circle $x^2 + y^2 = 4$.

Find the values of $f(3)$, $f'(1-i)$ and $f''(1-i)$. 8

OR

4 (a) If $w = f(z) = u + iv$ is an analytic function of $z = x + iy$ and $u - v = (x - y)(x^2 + 4xy + y^2)$. Find w in terms of z. 6

(b) Evaluate the following integral by Cauchy's integral formula :

$$\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$$

where C is the circle $|z| = 3$.

5

(c) Evaluate $\int_{1-i}^{2+i} (2x + 2iy + 3)\, dz$ along the path $x = t + 1$, $y = 2t^2 = 1$. 5

UNIT - V

5 (a) Find the residue of $f(z) = \frac{z^2 - 2z}{(z+1)^2 (z^2 + 4)}$ at all its poles in the finite plane. Hence evaluate $\int_C f(z)\, dz$ where C is the circle $|z| = 2$. 8

(b) Evaluate the integral by contour integration :

$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx \; (a > 0,\ b > 0).$$

8

OR

5 (a) Expand the following function is Laurent's series :

$$f(z) = \frac{1}{(z-1)(z-2)} \text{ for}$$

(i) $1 < |z| < 2$

(2) $|z| > 2$.

8

(b) Show by the method of residues, that

$$\int_0^{2\pi} \frac{d\theta}{(5 - 3\sin\theta)^2} = \frac{5\pi}{32}.$$

8
