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II. B.E. (III Semester)

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II. B.E. (Third Semester) Main/Back

EXAMINATION, December, 2005

(New Four Year Scheme)

(Branch : Mechanical Engineering and Automobile Engineering)

MATHEMATICS

Time : Three Hours

Maximum Marks : 80

Attempt any *Five* questions.

All questions carry equal marks.

1. Solve the following differential equations :

(a) $\frac{d^2y}{dx^2} - 2\tan x \frac{dy}{dx} + 4y = 12\, e^x \sec x.$ 8

(b) $\cos x \frac{d^2y}{dx^2} + \sin x \frac{dy}{dx} - 2y \cos^2 x = \cos^5 x.$ 8

2. (a) Use method of variation of parameter to solve :

$$\frac{d^2y}{dx^2} + 4y = \sec 2x.$$ 8

(b) Solve in series :

$$\left(1 - x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 12y = 0.$$ 8

3. (a) Solve :

(i) $x^2(y - z)\ p + y^2(z - x)\ q = z^2(x - y)$ 4

(ii) $z = px + qy + p^2q^2$. 4

(b) Use Charpit's method to solve :

$$p^2y = 2(z + xp + yq).$$ 8

4. (a) The ends A and B of a bar of length 'l' are kept at 0°C and 100°C respectively until steady state conditions prevail. If the temperature of A is then raised suddenly to 25°C while that of B is reduced to 75°C and kept so. Find the temperature distribution in the bar. 8

(b) Find the inverse laplace transform of :

(i) $\frac{s}{(s+1)^2(s^2+1)}$. 4

(ii) $\frac{s}{s^2+w^2}.e^{-as}$. 4

5. (a) Apply convolution theorem to obtain the inverse Laplace transform of :

$$\frac{s^2}{(s^2+4)(s^2+9)}.$$ 8

(b) Use Laplace transform method to solve :

$$\frac{\partial^2 y}{\partial t^2} = 9\frac{\partial^2 y}{\partial x^2}$$

subject to the conditions :

$y(0, t) = 0, \quad y(2, t) = 0$

$y(x, 0) = \sin 2\pi x - 2\sin 5\pi x$

and $y_t(x, 0) = 0$. 8

6. (a) Find half range cosine series to represent :

$f(x) = (x - 1)^2$ for $0 < x < 1$.

and hence show that :

$$\pi^2 = 8\left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots\right).$$ 6+2

(b) Obtain the expansion of y from the following table upto the third harmonic.

x	y
0	20
$\pi/3$	22
$2\pi/3$	28
π	–1
$4\pi/3$	–2
$5\pi/3$	15
2π	20

8

7. (a) Three boxes contain 1 white, 2 red and 3 black; 2 white, 3 red and 1 black; 3 white, 1 red and 2 black balls respectively. A box is chosen at random and two balls are drawn at random from it. These balls are one red and one white. Find the probability that these are drawn from :

(i) the first box,

(ii) the second box; and

(iii) the third box. 8

(b) Define the Binomial distribution and find its mean and variance. 8

8. (a) The frequency of accidents per shift in a factory is given below. Calculate the mean number of accidents per shift; find the corresponding Poisson distribution and compare with the actual observations.

Accidents per shift	Frequency
0	192
1	100
2	24
3	3
4	1

8

(b) (i) Write the probability density function for the normal distribution. 2

(ii) The distribution of weakly wages for 500 workers in a factory is approx. Normal with the mean and standard deviation of Rs. 750 and Rs. 150. Find the number of workers who receive weekly wages more than Rs. 900 and who receive weekly wages less than Rs. 450.

(Given : $P(0 < t < 1) = 0.3413$ and $P(0 < t < 2) = 0.4772$). 6