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II B.E.

4 CP 1 **Statistics. & Proba. Theo.**

II B.E. (IV Semester) (Main/Back) EXAMINATION, 2006

(New Four-Year Semester Scheme)

(Computer Engineering)

STATISTICS AND PROBABILITY THEORY

Time allowed : Three hours

Maximum marks : 80

Attempt any five questions. All questions carry equal marks.

1. (a) State and prove the Bayes theorem. 8

(b) Three groups of children contain respectively 3 girls and 1 boy, 2 girls and 2 boys, 1 girl and 3 boys. One child is selected at random from each group. Find the probability of selecting 1 girl and 2 boys. 8

$\frac{13}{32}$

2. (a) In a certain distribution, the first four moments about

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$\mu_2 = 19.75$

$\mu_3 = 35.75$

the point 4 are −1.5, 17, −30 and 108. Calculate β_1 and β_2 and state whether the distribution is leptokurtic or platykurtic. 8

(b) Given the joint probability density :

$$f(x, y) = \begin{cases} \frac{2}{3}(x+2y), & 0<x<1,\ 0<y<1 \\ 0, & \text{elsewhere} \end{cases}$$

Find :

(i) Marginal density of X and Y.

(ii) Conditional density of X given $Y = y$ and use it to evaluate :

$$P\left\{\frac{X \le \frac{1}{2}}{Y = \frac{1}{2}}\right\}$$ 8

3. (a) Let the random variable X assume the value r with probability-mass function :

$$P(X = r) = q^{r-1} p \quad r = 1, 2, 3, ...$$

Find the m.g.f. and hence the mean and variance. 5

(b) Compute the mean time to failure of a component of which the time T to failure follows a Weibul distribution with p.d.f. with α, β parameters given by :

$$f(t) = \alpha \beta t^{\beta-1} e^{-\alpha t^{\beta}}, \quad t > 0$$ 5

(c) Find mean and variance of the Binomial distribution. 6

4. (a) Define Poisson distribution.

Let λ and μK denote the mean and central Kth moment of a Poisson distribution respectively, obtain the following recurrence formula :

$$\mu_{K+1} = K\lambda\mu_{K-1} + \lambda \frac{d\mu K}{d\lambda}$$

and hence obtain β_1 and β_2 of the Poisson distribution. 8

(b) Define normal distribution.

The distribution of weekly wages for 500 workers in a factory is approximately normal with the mean and standard deviation of Rs. 75 and Rs. 15. Find the number of workers who receive weekly wages :

(i) more than Rs. 90

(ii) less than Rs. 45 4+4

5. (a) Define Poisson process. Prove that the sum of the two independent Poisson Process is a Poisson Process. 8

(b) In a railway marshalling yard, goods trains arrive at the rate of 30 trains per day. Assuming that the inter arrival time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes. Calculate the following :

(i) The mean queue size (line length)

(ii) The probability that the queue size exceeds 10.

(iii) If the input of trains increase to an 33 trains per day, what will be the change in (i) and (ii) ? 2+2+2+2

6. (a) A petrol pump has 2 pumps. The service times follows the exponential distribution with a mean of 4 minutes and vehicles arrive for service in Poisson fashion at the rate of 10 per hour.

(i) Find the probability that an arrival of a vehicle would have to wait.

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(ii) Find the expected percentage of ideal time for each petrol pump. 4+4

(b) Write a short note on Discrete-Parameter Birth death process. 8

7. (a) Define coefficient of correlation. Calculate the coefficient of correlation from the following data :

x	y
1	8
3	12
5	15
7	17
8	18
10	20

11.9

5

(b) For a bivariate distribution

$n = 18, \Sigma x^2 = 60, \Sigma y^2 = 96, \Sigma x = 12, \Sigma y = 18, \Sigma xy = 48.$

Find the equations of lines of regression and r. 6

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(c) Two products A and B have equal popularity. If their transition matrix be :

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} .9 & .1 \\ .5 & .5 \end{bmatrix} \end{array}$$

$P_A = 5/6$
$P_B = 1/6$

Find their probabilities in the steady state. 5

8. (a) Explain the method of least squares forfeiting of a second degree parabola. 6

(b) Find the mean and standard deviation of the exponential distribution whose p.d.f. is given by :

$$f(x) = \lambda e^{-\lambda k}, \ x \geq 0$$

$$= 0, \text{ otherwise}$$

Also show that for the exponential distribution given by :

$$dp = ae^{-x/c}\, dx, \quad 0 < x < \infty$$

$C > 0$, a being a constant, the mean and standard deviation are each equal to C. 5+5