B

898

II B.E.

4 EC 1 **Advanced Maths.**

II B.E. (IV Semester) (Main/Back) EXAMINATION, 2006

(New Four-Year Semester Scheme)

(Electronics and Communications Engineering)

ADVANCED MATHEMATICS

Time allowed : Three hours

Maximum marks : 80

Attempt any five questions. All questions carry equal marks.

1. (a) Show that the function :

$$f(z) = \frac{x^2 y^5 (x + iy)}{\left(x^4 + y^{10}\right)}, \quad z \neq 0 \text{ and } f(0) = 0$$

is not analytic at $z = 0$, although Cauchy-Riemann equations are satisfied at that point. 8

PTO

(b) Define the conformal transformation and under the transformation :

$$W = \frac{1}{z},$$

(i) Find the image of $|z - 2i| = 2$ $\quad 4v + 1 = 0$

(ii) Show that the image of the hyperbola $x^2 - y^2 = 1$ is the lemniscate $\rho^2 = \cos 2\phi$.

2+3+3

2. (a) Evaluate :

$$\int_{1-i}^{2+i} (2x + iy + 1)\, dz \qquad 4 + \frac{25}{3} i$$

along the path $x = t + 1,\ y = 2t^2 + 1$.

4

(b) State and prove Cauchy's Integral Theorem.

6

(c) Evaluate the following integral by using Cauchy's integral formula :

$$\frac{1}{2\pi i} \int_C \frac{e^{tz}}{z^2 + 1}\, dz,\ t > 0 \qquad \sin t \quad t > 0$$

where C is the circle $|z| = 3$.

6

3. (a) Expand :

$$f(z) = \frac{1}{(z+1)(z+3)} \qquad = \frac{1}{2}\left[\frac{1}{z+1} - \frac{1}{z+3}\right]$$

in a Laurent's series valid for :

(i) $|z| < 1$

(ii) $1 < |z| < 3$

(iii) $|z| > 3$

6

(b) Find the poles of the function :

$$f(z) = \frac{e^z}{z(z+1)^2}.$$

Also, find the order of each pole and residue at it.

6

(c) Evaluate :

$$\int_C \frac{12z - 7}{(z-1)^2 (2z+3)}\, dz \qquad 2\pi i\, [\text{Residue at } z=1] = 4\pi i$$

where C is the circle : $|z + i| = \sqrt{3}$.

4

Evaluate the following by contour integration :

$f(z) = \frac{z}{(3z^2 - 10iz - 3)^2}$ $z =$

(i) $\int_0^{2\pi} \frac{d\theta}{(5-3\sin\theta)^2}$ $= \frac{5\pi}{32}$

(ii) $\int_0^{\infty} \frac{\sin mx}{x}\, dx$ when $m > 0$ $= \pi/2$ 8+8

5. (a) Find the path on which a particle, in the absence of friction, will slide from one fixed point to another point in the shortest time under the action of gravity. 8

$x = b(\phi - \sin\phi)$ $y = b(1-\cos\phi)$

(b) Find a function $y(x)$ for which

$$\int_0^1 \left[x^2 - (y')^2\right] dx \text{ is stationary,}$$

given that $\int_0^1 y^2 dx = 2$, $y(0) = 0$, $y(1) = 0$. 8

$y = c_2 \sin n\pi$

6. (a) Find the Laplace transforms of the following :

(i) $(x+2)^2 e^x$ $\frac{2}{(p-1)^3} + \frac{4}{p-1} + \frac{4}{(p-1)^2}$

(ii) $\frac{\sin x}{x}$ $\pi/2 - \tan^{-1}(s) = \cot^{-1}(s)$ 3+3

(b) Find $L^{-1}\left\{\frac{4p+5}{(p-1)^2(p+2)}\right\}$. 4

$\frac{1}{3}e^t + 3te^t - \frac{1}{3}e^{-2t}$

(c) Find the inverse Laplace transform with the help of convolution theorem :

$$\frac{p}{\left(p^2 + a^2\right)^2}$$ 6

$\frac{1}{2a} t \sin at$

7. (a) Use Laplace transform theory to solve the following equation :

$$\left(D^2 + 2D + 1\right) y = t$$

given that $y(0) = -3$ and $y(1) = -1$. 6

(b) Find :

$$L\left\{\frac{\partial^2 u}{\partial t^2}\right\}$$

where $u(x, t)$ is a function of two independent variables for $a \le x \le b$, $t > 0$. 4

(c) Solve :

$$\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2}$$

where $u\left(\frac{\pi}{2}, t\right) = 0$, $\left(\frac{\partial u}{\partial x}\right)_{x=0} = 0$, $u(x, 0) = 30\cos 5x$. 6

8. (a) Find $f(x)$ if its Fourier sine transform is :

$$\frac{e^{ap}}{p} \qquad \sqrt{\frac{2}{\pi}}\tan^{-1}\left(\frac{x}{a}\right)$$

Hence deduce $F_s^{-1}\left(\frac{1}{p}\right)$. $\sqrt{\frac{\pi}{2}}$ 8

(b) Solve :

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2} \; ; x > 0, t > 0$$

Subject to the conditions :

(i) $V = 0$ when $x = 0, t > 0$

(ii) $V = \begin{cases} 1, & 0 < x < 1 \text{ when } t = 0 \\ 0, & x \geq 1 \end{cases}$

(iii) $V(x, t)$ bounded 8