Exam cell 08/07/2010

4E2137

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B.Tech. IVth Semester (Main/Back) Examination, June - 2010
Electronics & Communication
Mathematics - IV
Common 4ECI, 4EI6.3, 4AI1, 4BM6.3, 4CRE5

Time : 3 Hours **Maximum Marks : 80**

Min. Passing Marks : 24

Instructions to Candidates:

*Attempt any **five questions** selecting **one question** from **each unit**. All questions carry **equal marks**. (Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.)*

Unit - I

1. a) From the following table of values of x and $f(x)$, determine

(i) $f(0.23)$, (ii) $f(0.29)$.

x :	0.20	0.22	0.24	0.26	0.28	0.30
$f(x)$:	1.6596	1.6698	1.6804	1.6912	1.7024	1.7139

State the formulae used. **(8)**

b) Apply Lagrange's formula to find $f(x)$ from the following data :

x :	0	1	4	5
$f(x)$:	4	3	24	39

(8)

OR

a) State and use the Stirling formula to find u_{32} from the following table:

$u_{20}=14.035$, $u_{25}=13.674$, $u_{30}=13.257$, $u_{35}=12.734$, $u_{40}=12.089$, $u_{45}=11.309$. **(8)**

b) Define the operators δ and μ, and prove that

$$\delta[f(x)\,g(x)]=\mu[f(x)]\,\delta[g(x)]+\mu[g(x)]\,\delta[f(x)].$$ **(8)**

Unit - II

2. a) Compute the values of : $\int_0^6 \frac{dx}{1+x^2}$ by the

i) Simpson's $\frac{1}{3}$ rule

ii) Simpson's $\frac{3}{8}$ rule and

iii) Trapezoidal rule and compare result with the exact value of the integral. **(8)**

b) Employ Runge-Kutta fourth order method to obtain $y(0.2), y(0.4), y(0.6)$ from

$\frac{dy}{dx} = 1 + y^2$, with $x = 0, y = 0$. Take $h = 0.2$. **(8)**

OR

a) Given : $\frac{dy}{dx} = x^2 + y$, $y(0) = 1$ determine $y(0.02)$, $y(0.04)$, $y(0.06)$, using the modified method of Euler. **(8)**

b) A rod is rotating in a plane. The following table gives the angle θ (in radians) through which the rod has turned for various values of time t (in seconds) :

t :	0	0.2	0.4	0.6	0.8	1.0	1.2
θ :	0	0.12	0.49	1.12	2.02	3.20	4.67

Calculate the angular velocity and the angular acceleration of the rod when $t = 0.6$ seconds. **(8)**

Unit - III

3. a) Find $J_{1/2}(x)$ and $J_{-1/2}(x)$ for Bessel functions. **(8)**

b) State and prove Rodrigue's formula for Legendre polynomials. **(8)**

OR

a) By putting $u = yx^{3/2}$, transform the differential equation.

$$x\frac{d^2u}{dx^2} - 2\frac{du}{dx} + xu = 0$$

into Bessel's equation and solve it. **(8)**

b) For Legendre polynomials prove the following orthogonality property :

i) $\int_{-1}^{1} P_m(x)\, P_n(x)\, dx = 0$, for $m \neq n$.

ii) $\int_{-1}^{1} [P_n(x)]^2\, dx = \frac{2}{2n+1}$, for $m = n$. **(8)**

Unit - IV

4. a) A committee consists of 9 students. Two of whom are from first year, three from second year and four from third year. Three students are to be removed at random. What is the chance that **(8)**

   i) The three students belong to different classes,

   ii) Two belong to the same class and third to a different class,

   iii) All three belong to the same class?

   b) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D. of the distribution.

   [Given $P = 0.19, 0.23, 0.42$ for $t = 0.50, 0.60, 1.4$ respectively] **(8)**

OR

   a) Fit a poisson distribution to the following data for 10 Prussian Army Corps over 20 years, which gives the frequency of the number of deaths from the kick of a horse per army corps per year; and calculate the theoretical frequencies.

No. of deaths :	0	1	2	3	4
Frequency :	109	65	22	3	1

(8)

   b) Two random variables have the least square regression lines with equations :

   $3x + 2y - 26 = 0$ and $6x + y - 31 = 0$.

   Find the mean values and the coefficient of correlation between x and y. **(8)**

Unit - V

5. a) Prove that the shortest distance between two given points in a plane is a straight line. **(8)**

   b) Find the extremals of the functional

$$V[y(x), z(x)] = \int_0^{\pi/2} \left[(y')^2 + (z')^2 + 2yz\right] dx, \text{ where}$$

$$y(0) = 0,\ y\left(\frac{\pi}{2}\right) = 1,\ z(0) = 0,\ z\left(\frac{\pi}{2}\right) = -1.$$

(8)

OR

   a) Find the extremals of the isoperimetric problem

$$V[y(x)] = \int_0^1 \left[(y')^2 + x^2\right] dx$$

given $\int_0^1 y^2\, dx = 2,\quad y(0) = 0,\quad y(1) = 0$. **(8)**

   b) Prove that the sphere is the solid figure of revolution which, for a given surface area, has maximum volume. **(8)**