

Semester: 2nd  
Subject: BME

Branch: EE  
Submitted by: Deepak Sharma

Q1. Define Mechanical Engineering & its scope

Ans:- Mechanical Engineering is the branch of Engineering that involves design, production and operation of machinery with the usage of heat and mechanical power. In mechanical engineering deals with anything that moves from the tiniest micro-particle to the largest spacecraft & even the human body. It is a diverse subject that derives its breadth from the need to design and manufacturing everything from small individual parts and devices. The role of a mechanical engineer is to develop a product from an idea to the market place. The ~~most~~ mechanical engineer needs to understand the forces and the thermal environment that a product and its parts or its ~~sys~~ subsystem will encounter. He also design them for functionality, aesthetics, and ability to withstand the forces and the thermal environment that a product and its parts or its subsystem will encounter.

Mechanical Engineer plays a central role in industries such as automotive, aerospace, biotechnology, computers and electronics, MEMS, energy conversion, HVAC etc. Mechanical engineering allows learning about materials, solid & fluid mechanics, thermodynamics, heat transfer design & manufacturing to understand mechanical system. Specialized mechanical engineering subjects include biomechanics, energy conversion, laser-assisted material processing, combustion, MEMS, fracture mechanics, nanomechanics, tribology and vibration.

Mechanical engineering applies engineering, physics and material science principle to design, analyze manufacturing and maintain mechanical system.



The mechanical Engineering field requires an understanding of core areas including mechanics, kinematics, thermodynamics, material science, structural analysis and electricity.

In addition to these mechanical engineers use tools such as computer-aided design (CAD), and product life cycle management to design and analyze manufacturing plants, industrial equipment and machinery heating and cooling systems, transport system, aircraft, watercraft, robotics etc.

Q2. Describe the laws of thermodynamics in detail:

Ans:- Thermodynamics is a branch of science that deals with heat and temperature. It is an application of thermodynamics and other sciences, which involving designing, building and maintaining things that are related to heating and cooling.

### Laws of Thermodynamics:-

#### 1) Zeroth Laws of Thermodynamics:-

Two thermal system are in thermal equilibrium with a third system, they are in thermal equilibrium with each other.

This law helps the notation of temperature.

2) First law of thermodynamics also known as law of conservation of energy, states that energy can neither be created nor destroyed. energy can only be transferred or changed from one form to another.

For example:- turning on a light would seem to produce energy; however it is electrical energy that is converted

A way of expressing the 1<sup>st</sup> law of thermodynamics is that any change in the internal energy ( $\Delta E$ ) of a system is given by the sum of heat ( $q$ ) that flows across its boundaries and the work ( $w$ ) done on the system by the surroundings:-

$$\Delta E = q + w$$



This law says that there are two kinds of processes, heat and work, that can lead to a change in the internal energy of a system. Since both heat and work can be measured and quantified this is the same as saying that any change in the energy of a system must result in a corresponding change in the energy of the surroundings outside of the system. If heat flows into a system or the surroundings do work on it, the internal energy increases and the sign of  $q$  and  $w$  are positive. Conversely heat flow out of the system or work done by the system (on the surroundings) will be at the expense of the internal energy, and  $q$  and  $w$  will therefore be -ive.

$$dQ = \frac{dw}{J} \quad \begin{array}{l} dQ = \text{change in heat supplied} \\ dw = \text{change in work} \\ J = \text{Joule} \end{array}$$

### 3) The Second Law of Thermodynamics :-

The 2<sup>nd</sup> law of thermodynamics says that entropy of any isolated system always increases. Isolated system spontaneously evolve towards towards thermal equilibrium. The state of maximum entropy of the system. more simply put: the entropy of the universe only increases and never decreases.

Entropy:- There are 3 important E's in the study of the thermodynamics energy, equilibrium and entropy. Entropy is the extensive property of the system and its unit of measurement is J/K. Entropy is heat or energy change per degree Kelvin temperature. Entropy is denoted by 'S' and specific entropy is 's'.

The entropy plays central role in the study of thermodynamics and it has been introduced via the concept of the working of the heat engine. The entropy of the system is <sup>not</sup> measured in absolute terms rather than it is measured in relative terms. The entropy of the system is measured in terms of the changes the system has undergone from the previous state to the final state. Thus the entropy is always measured as the change in entropy of the system ~~not~~ denoted by  $\Delta S$  and not merely S.

The process during which the entropy of the system remain constant is called as isentropic process.



During isentropic process the value of entropy of the system remains ~~at~~ initial and final state remains constant. Thus during isentropic process the value of  $\Delta S = 0$ . (9)

In an isothermal process, the change in entropy is the change in heat ( $Q$ ) divided by the absolute Temperature ( $T$ )

$$\Delta S = \frac{dQ}{T}$$

Entropy is considered to be an extensive property of matter that is expressed in terms of energy divided by temperature. SI units of entropy are J/K.

Q9) The third law of thermodynamics:- The third law of thermodynamics states that the entropy of a system approaches a constant value as the temperature approaches absolute zero. The entropy of a system at absolute zero is typically zero and in all cases is determined only by the no. of different ground states it has. Specifically, the entropy of a pure crystalline substance at zero absolute zero temperature is zero.

Q3: Define machine design and write its importance in mechanical engineering.

Ans:- The word design as defined by the following way:-

- 1) A drawing or pattern showing how something is to be made.
- 2) The art of making such drawings or patterns;
- 3) The arrangement of parts in any man-made product, such as a machine or work of art,
- 4) A decorative pattern, esp. one that is not repeated.
- 5) A plan in the mind.

The word design is also used as a verb with the following meaning:

- 1) To make a drawing or pattern of, develop and draw the plans for;

- 2) To plan or develop for a certain purpose or use.

Mechanical machine design is the branch of engineering design, that can lead to the formation of the entirely new machine or it can lead



to improvement of the existing machine. To understand what exactly machine design ~~des~~ is, let's consider the example of the gear box of the car. The gear box transmits the power of the engine to the wheels of the vehicle. The gear box consists of gears which are subjected to not only motion but also the load of vehicle.

For the gears to run at desired speeds and take desired loads it is important that they should be designed.

During designing various calculations are performed considering desired speeds and load and finally the gear of particular material and specific dimensions that can take all loads and that can be manufactured at least possible cost giving optimum performance is designed. In similar fashion all the components of the car, including engine have to be designed so that they optimally meet all the functional requirements at lowest possible cost. This whole process of designing is called as machine design or mechanical design. Machine design can lead to the formation of the entirely new machine or it can lead to up-gradation or improvement of the existing machine.

Q.4: Discuss about industrial engineering and its various scope.

Ans:- It is a branch of engineering which deals with the optimization of complex processes, systems or organizations. Industrial engineers work to eliminate waste of time, money, materials, person hours, machine time, energy and other resources that do not generate value. According to the Institute of Industrial and System Engineers, they create engineering processes and systems that improve quality and productivity. Industrial engineering is concerned with the development, improvement and implementation of integrated systems of people, money, knowledge, information, equipment, energy, materials, analysis and synthesis, as well as the mathematical, physical and social sciences together with the principles and methods of engineering design to specify, predict



evaluate the results to obtained from such system or processes. While industrial engineering is a long standing engineering discipline subject to professional engineering licensure in most jurisdictions, its underlying concepts, overlap considerably with certain business-oriented disciplines such as operation management.

Depending on the sub-specialties involved, industrial engineering may also be known as, or overlap with, operations research systems engineering, manufacturing engineering, production engineering, management science, management engineering, ergonomics or human factors engineering, safety engineering, or other depending on the viewpoint or motives of the user.

While originally applied to manufacturing, the use of industrial in "industrial engineering" can be somewhat leading, since it has grown to encompass any methodical or quantitative approach to optimizing how a process, system, or organization operates. Some engineering universities and educational agencies around the world have changed the term "industrial" to broader terms such as "production" or "system", leading to the typical extensions noted above.

The various topics concerning industrial engineering include:

Process engineering, design, operation, control, and optimization of chemical, physical, and biological processes.

The industrial engineering involves in the following fields:-

- system engineering, safety engineering, data science, machine learning, Analytics and data mining, cost engineering, value engineering, quality engineering, Project Management, management engineering, ergonomics, operation research and management, job design, financial engineering, facility management, Engineering design process, logistics etc.

Examples:- Industrial engineering include flow process charting process mapping, designing an assembly workstation



Modern industrial engineers typically use predetermined motion time system, computer simulation along with extensive mathematically tools for modeling such as mathematical optimization and queuing theory and computational methods for system analysis evaluation, and optimization. Industrial engineers also use the tools of data science and machine learning in their work owing to the strong relatedness of these disciplines with the field and the similar technical background required of Industrial Engineers.

Q5. Define the steam generator and give its classification?

Steam generator is a boiler which is basically a closed vessel into which water is heated until the water is converted into steam at required pressure. Fuel is burnt in a furnace and hot gases are produced. The hot gases come in contact with water vessel where the heat of these hot gases transfer to the water and consequently steam is produced in the boiler. Then this steam is piped to the turbine of thermal power plant. There are different types of boilers utilized for different purposes like running a production unit, sanitizing some area, sterilizing equipment, to warm up the surrounding etc. A typical efficiency of steam boiler is 80% to 88% due to some losses like incomplete combustion, radiating loss occurs from steam boiler surrounding wall, defective combustion gas etc.

Boiler classification:- Boilers can be classified as follows:-

1) According to the flow of water and hot gases - fire tube and water tube boilers. In fire tube boilers, hot gases pass tubes which are surrounded with water.

Ex:- Vertical, Cochran, Lancashire etc.

In water tube boilers, water circulates through a large number of tubes and hot gases pass around them.

Ex:- Babcock and Wilcox boiler.



2) According to the axis of the shell:- The boilers can also be classified according to the ~~axis~~ <sup>axis</sup> of the shell. When the axis of the ~~shell~~ <sup>boiler</sup> is vertical then the boiler is known as vertical boiler while the boiler shell axis is horizontal then the boiler is known as horizontal boiler.

3) According to location or furnace position:- externally and internally fired boilers. In internally fired boilers, the furnace forms an integral part of the boiler structure. Ex:- Locomotive, Scotch marine boilers etc.

Externally fired boilers have a separate furnace built outside the boiler shell and usually below it. The (HRT) boiler is ~~probably~~ probably the most widely example.

4) According to the application:- stationary and mobile boilers. A stationary boiler is one, which is installed permanently on a land. A marine boiler is a mobile boiler meant for ocean cargo and passengers ships with an inherent fast steaming capacity.

5) According to steam pressure - the boilers are classified as low pressure, medium pressure and high pressure boiler.

Low pressure boiler:- A boiler which produces steam at a pressure of 15-20 bar is called a low pressure boiler. This steam is used for heating process.

Medium pressure boiler:- If a boiler working between 20 to 80 bars steam pressure then it is known as medium pressure boiler. This steam is for power generation and heating process.

High pressure boiler:- It produces steam at pressure which is above 80 bar.



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**I-MID TERM PAPER ANSWER SHEET**

**Sem:-II**  
**Subject:-CP-II**

**Branch:-CS/EE/ME/CE**  
**Submitted by:-Pawan k. Jhajharia**

**Q.1** What is Array? What are advantages and disadvantages of Array? Explain different operations that can be performed on Array.

**Ans.** Array is a collection of similar data types under a single variable.

**Advantages:-**

- It is a better and convenient way of storing the data of same datatype with same size.
- It allows us to store known number of elements in it.
- It allocates memory in contiguous memory locations for its elements. It does not allocate any extra space/ memory for its elements. Hence there is no memory overflow or shortage of memory in arrays.
- Iterating the arrays using their index is faster compared to any other methods like linked list etc.
- It allows to store the elements in any dimensional array - supports multidimensional array.

**Disadvantages**

- It allows us to enter only fixed number of elements into it. We cannot alter the size of the array once array is declared. Hence if we need to insert more number of records than declared then it is not possible. We should know array size at the compile time itself.
- Inserting and deleting the records from the array would be costly since we add / delete the elements from the array, we need to manage memory space too.
- It does not verify the indexes while compiling the array. In case there is any indexes pointed which is more than the dimension specified, then we will get run time errors rather than identifying them at compile time.

**Basic Operations**

Following are the basic operations supported by an array.

- **Traverse** – print all the array elements one by one.



- **Insertion** – Adds an element at the given index.
- **Deletion** – Deletes an element at the given index.
- **Search** – Searches an element using the given index or by the value.
- **Update** – Updates an element at the given index.

**Q2. Write a Program to Swap two numbers by using:-**

(i) Call By Value

```
#include<stdio.h>
#include<conio.h>

int swap(int , int);                // Declaration of function

main( )
{
    int a = 10, b = 20 ;            // call by value
    swap(a,b);                      // a and b are
    actual parameters
    printf ( "\na = %d b = %d", a, b ) ;
    getch();
}

int swap( int x, int y )            // x and y are formal
parameters
{
    int t ;
    t = x ;
    x = y ;
    y = t ;
    printf ( "\nx = %d y = %d", x, y ) ;
}
```

(ii) Call By Address

```
#include<stdio.h>

#include<conio.h>

void main()

{

int a,b,t;

clrscr();

void swap(int *,int *);
```



```

printf(" Enter two numbers :");

scanf("%d%d",&a,&b);

printf("\n Before Exchange a=%d , b=%d ",a,b);

swap(&a,&b);

printf("\n After Exchange a=%d , b=%d ",a,b);

getch();

}

void swap(int *x,int *y)

{

int t;

t=*x

x=*y;

y=t;

}

```

**Q.3 (a)** What is Pointer? Explain Pointers with Example.

Sol:-

(i)Pointers in C language is a variable that stores/points the address of another variable. A Pointer in C is used to allocate memory dynamically i.e. at run time. The pointer variable might be belonging to any of the data type such as int, float, char, double, short etc.

(ii)Pointer Syntax : data\_type \*var\_name; Example : int \*p; char \*p;

(iii)Where, \* is used to denote that "p" is pointer variable and not a normal variable.

**Key points to remember about pointers in C:**

- Normal variable stores the value whereas pointer variable stores the address of the variable.
- The content of the C pointer always be a whole number i.e. address.
- Always C pointer is initialized to null, i.e. int \*p = null.
- The value of null pointer is 0.
- & symbol is used to get the address of the variable.
- \* symbol is used to get the value of the variable that the pointer is pointing to.
- If a pointer in C is assigned to NULL, it means it is pointing to nothing.



- Two pointers can be subtracted to know how many elements are available between these two pointers.

Example- #include <stdio.h>

```
int main()
{
    int *ptr, q;
    q = 50;
    /* address of q is assigned to ptr */
    ptr = &q;
    /* display q's value using ptr variable */
    printf("%d", *ptr);
    return 0;
}
```

(b) Write a Program to calculate sum of N values in given array.

Sol:- /\* CPP Program to find sum of elements  
in a given array \*/

#include <bits/stdc++.h>

// function to return sum of elements

// in an array of size n

int sum(int arr[], int n)

{

int sum = 0; // initialize sum

// Iterate through all elements

// and add them to sum

for (int i = 0; i < n; i++)

sum += arr[i];

return sum;

}

int main()

{

int arr[] = {12, 3, 4, 15};

int n = sizeof(arr) / sizeof(arr[0]);

printf("Sum of given array is %d", sum(arr, n));

return 0;

}

**Q.4 (a)** Write a program to implement Linear Search method.

**Sol:-** This C Program implements linear search. Linear search is also called as sequential search.

Linear search is a method for finding a particular value in a list, that consists of checking every one of its elements, one at a time and in sequence, until the desired one is found.

Here is source code of the C program to implement linear search. The C program is successfully compiled and run on a Linux system. The program output is also shown below.

```
/* C program to input N numbers and store them in an array.
 * Do a linear search for a given key and report success or failure. */
#include <stdio.h>
void main()
{
    int array[10];
    int i, num, keynum, found = 0;
    printf("Enter the value of num \n");
    scanf("%d", &num);
    printf("Enter the elements one by one \n");
    for (i = 0; i < num; i++)
    {
        scanf("%d", &array[i]);
    }
    printf("Input array is \n");
    for (i = 0; i < num; i++)
    {
        printf("%d/n", array[i]);
    }
    printf("Enter the element to be searched \n");
    scanf("%d", &keynum);
    /* Linear search begins */
    for (i = 0; i < num ; i++)
    {
        if (keynum == array[i] )
        {
            found = 1;
            break; }}
}
```



```

if (found == 1)
    printf("Element is present in the array\n");
else
    printf("Element is not present in the array\n");
}

```

(b) Write a program to arrange values in particular order (Sorting).

**Sol:** - C program to sort elements of array in ascending order

```

*/

#include <stdio.h>
#define MAX_SIZE 100    // Maximum array size

int main()
{
    int arr[MAX_SIZE];
    int size;
    int i, j, temp;

    /* Input size of array */
    printf("Enter size of array: ");
    scanf("%d", &size);

    /* Input elements in array */
    printf("Enter elements in array: ");
    for(i=0; i<size; i++)
    {
        scanf("%d", &arr[i]);
    }

    for(i=0; i<size; i++)
    {
        /*
         * Place currently selected element array[i]
         * to its correct place.
         */
        for(j=i+1; j<size; j++)
        {
            /*
             * Swap if currently selected array element
             * is not at its correct position.
             */
            if(arr[i] > arr[j])
            {
                temp      = arr[i];
                arr[i] = arr[j];
                arr[j] = temp;
            }
        }
    }

    /* Print the sorted array */
}

```

```

    printf("\nElements of array in ascending order: ");
    for(i=0; i<size; i++)
    {
        printf("%d\t", arr[i]);
    }

    return 0;
}

```

**Q.5** How Two-dimensional (2-d Array) stored in memory. Write a program to perform multiplication between two matrices (matrix should be input by user)

**Sol:-Initialization of 2D Array**

There are two ways to initialize a two Dimensional arrays during declaration.

```

int disp[2][4] = {
    {10, 11, 12, 13},
    {14, 15, 16, 17}
};

```

OR

```

int disp[2][4] = { 10, 11, 12, 13, 14, 15, 16, 17};

```

Although both the above declarations are valid, I recommend you to use the first method as it is more readable, because you can visualize the rows and columns of 2d array in this method.

Things that you must consider while initializing a 2D array

We already know, when we initialize a normal array (or you can say one dimensional array) during declaration, we need not to specify the size of it. However that's not the case with 2D array, you must always specify the second dimension even if you are specifying elements during the declaration. Let's understand this with the help of few examples –

```

/* Valid declaration*/

```

```

int abc[2][2] = { 1, 2, 3 ,4 }

```

```

/* Valid declaration*/

```

```

int abc[][2] = { 1, 2, 3 ,4 }

```

```

/* Invalid declaration – you must specify second dimension*/

```



```
int abc[][] = { 1, 2, 3 ,4 }
```

```
/* Invalid because of the same reason mentioned above*/
```

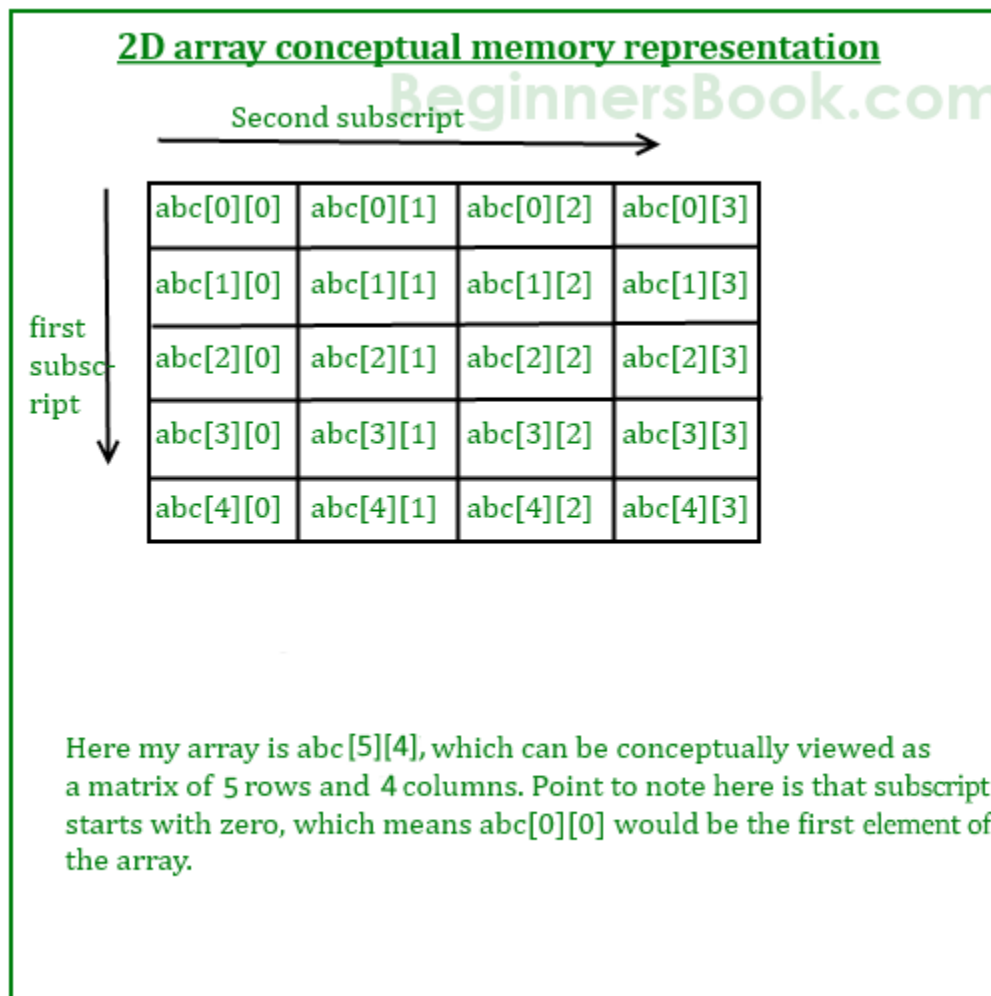
```
int abc[2][] = { 1, 2, 3 ,4 }
```

## How to store user input data into 2D array

We can calculate how many elements a two dimensional array can have by using this formula:

The array `arr[n1][n2]` can have  $n1 * n2$  elements. The array that we have in the example below is having the dimensions 5 and 4. These dimensions are known as subscripts. So this array has first subscript value as 5 and second subscript value as 4.

So the array `abc[5][4]` can have  $5 * 4 = 20$  elements.



abc[0][1]	abc[0][2]	abc[0][3]	abc[1][0]	abc[1][1]	.... ..	abc[4][2]	abc[4][3]
82206	82210	82214	82218	82222		82274	82278

memory locations for the array elements

Array is of integer type so each element would use 4 bytes that's the reason there is a difference of 4 in element's addresses.

The addresses are generally represented in hex. This diagram shows them in integer just to show you that the elements are stored in contiguous locations, so that you can understand that the address difference between each element is equal to the size of one element(int size 4). For better understanding see the program below.

### Actual memory representation of a 2D array

```
#include <stdio.h>

int main()
{
    int a[10][10], b[10][10], result[10][10], r1, c1, r2, c2, i, j, k;
    printf("Enter rows and column for first matrix: ");
    scanf("%d %d", &r1, &c1);
    printf("Enter rows and column for second matrix: ");
    scanf("%d %d", &r2, &c2);

    // Column of first matrix should be equal to column of second matrix and
    while (c1 != r2)
    {
        printf("Error! column of first matrix not equal to row of second.\n\n");
        printf("Enter rows and column for first matrix: ");
        scanf("%d %d", &r1, &c1);
        printf("Enter rows and column for second matrix: ");
        scanf("%d %d", &r2, &c2);
    }
}
```



```
// Storing elements of first matrix.
printf("\nEnter elements of matrix 1:\n");
for(i=0; i<r1; ++i)
    for(j=0; j<c1; ++j)
    {
        printf("Enter elements a%d%d: ",i+1, j+1);
        scanf("%d", &a[i][j]);
    }
```

```
// Storing elements of second matrix.
printf("\nEnter elements of matrix 2:\n");
for(i=0; i<r2; ++i)
    for(j=0; j<c2; ++j)
    {
        printf("Enter elements b%d%d: ",i+1, j+1);
        scanf("%d",&b[i][j]);
    }
```

```
// Initializing all elements of result matrix to 0
for(i=0; i<r1; ++i)
    for(j=0; j<c2; ++j)
    {
        result[i][j] = 0;
    }
```

```
// Multiplying matrices a and b and
// storing result in result matrix
for(i=0; i<r1; ++i)
    for(j=0; j<c2; ++j)
        for(k=0; k<c1; ++k)
        {
            result[i][j]+=a[i][k]*b[k][j];
        }
```

```
// Displaying the result
printf("\nOutput Matrix:\n");
for(i=0; i<r1; ++i)
    for(j=0; j<c2; ++j)
    {
        printf("%d ", result[i][j]);
        if(j == c2-1)
            printf("\n\n");
    }
return 0;
}
```

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**I-Mid Term Examination Session 2017-2018**

**B.Tech ...1st... Year ...2nd... Semester**

**Branch: common for all branches**

**Time: 10:00 A.M.-11.30A.M.**

**Date: 08/03/18**

**Subject: EEE**

**Subject Code: 105**

**Max. Marks: 20**

Note: Attempt any four questions out of five questions.

**Q.1 a) write statement of superposition theorems.**

ANS: Superposition theorem is based on the concept of linearity between the response and excitation of an electrical circuit. It states that the response in a particular branch of a linear circuit when multiple independent sources are acting at the same time is equivalent to the sum of the responses due to each independent source acting at a time.

In this method, we will consider only one independent source at a time. So, we have to eliminate the remaining independent sources from the circuit. We can eliminate the voltage sources by shorting their two terminals and similarly, the current sources by opening their two terminals.

Therefore, we need to find the response in a particular branch 'n' times if there are 'n' independent sources. The response in a particular branch could be either current flowing through that branch or voltage across that branch.

**Procedure of Superposition Theorem**

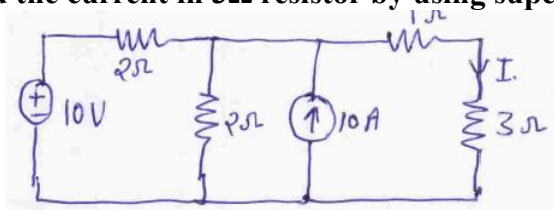
Follow these steps in order to find the response in a particular branch using superposition theorem.

Step 1 – Find the response in a particular branch by considering one independent source and eliminating the remaining independent sources present in the network.

Step 2 – Repeat Step 1 for all independent sources present in the network.

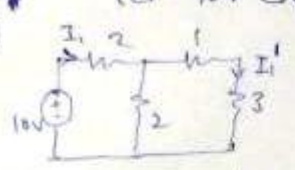
Step 3 – Add all the responses in order to get the overall response in a particular branch when all independent sources are present in the network.

**b) Find the current in  $3\Omega$  resistor by using superposition theorems.**





ANS. (b) CASE-1 (let 10V source are ON)



$R_{AB} = 2 \parallel (1+3)$

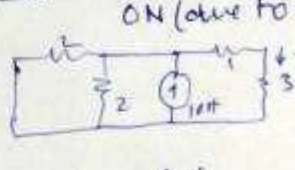
$$I_1 = \frac{10}{R_{AB}} = \frac{10}{10/3} = 3A$$

$$R_{AB} = 2 + 2 \parallel (1+3)$$

$$I_1' = \frac{\frac{1}{4}}{\frac{1}{2} + \frac{1}{4}} \cdot I_1 = \frac{6}{6} \text{ Amp}$$

$[I_1' = 1 \text{ Amp}]$

CASE-2 (let current source are ON (due to 10A))



$$I_1'' = \frac{6 \cdot \frac{1}{4} \times 10}{\frac{1}{2} + \frac{1}{4}} = \frac{94}{6+4+4}$$

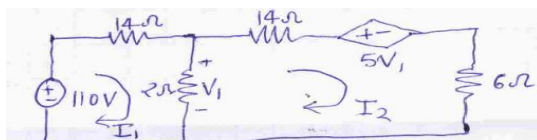
$$I_1'' = 2A$$

Now  $I = I_1' + I_1''$

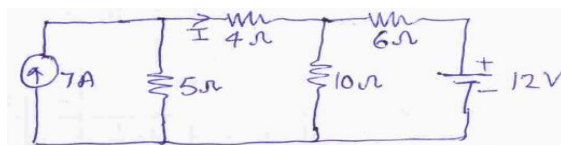
$$= 1 + 2 = 3 \text{ Amp}$$

ANS:

Q.2 a) finds current  $I_1$  &  $I_2$  by using Mesh analysis.



b) Find the current through  $4\Omega$  resistor by using Node Method



ANS 2

ANS. (3)  
b7

apply Node method at Node  $V_x$   
 $-7 + \frac{V_x}{5} + \frac{V_x - V_y}{4} = 0$  — (1)

apply Node at  $V_y$   
 $\frac{V_y - V_x}{4} + \frac{V_y}{10} + \frac{V_y - 12}{6} = 0$  — (2)

Calculate  $V_x$  &  $V_y$

From (1)  
 $140 = 9V_x - 5V_y$  — (3)

From (2)  
 $15V_y - 15V_x + 6V_y + 10V_y - 120 = 0$   
 $-15V_x + 31V_y = 120$  — (4)

From eq<sup>n</sup> (3) & (4) calculate  $V_x$  &  $V_y$

Now current in  $4\Omega$  Resistor by Node

$$I_{4\Omega} = \frac{V_x - V_y}{4}$$

$V_y = \frac{1290}{204}$   
 $V_x = \frac{35010}{1836}$

ANS. (2)  
(a)

apply mesh in loop abefg

$110 - 14I_1 - 2(I_1 - I_2) = 0$  — (1) From mesh abefg

$14I_2 - 5V_1 - 6I_2 - 2(I_2 - I_1) = 0$  — (2) From mesh bcdeb

&  $V_1 = 2(I_1 - I_2)$  — (3)

calculate  $I_1$  &  $I_2$  From given equations

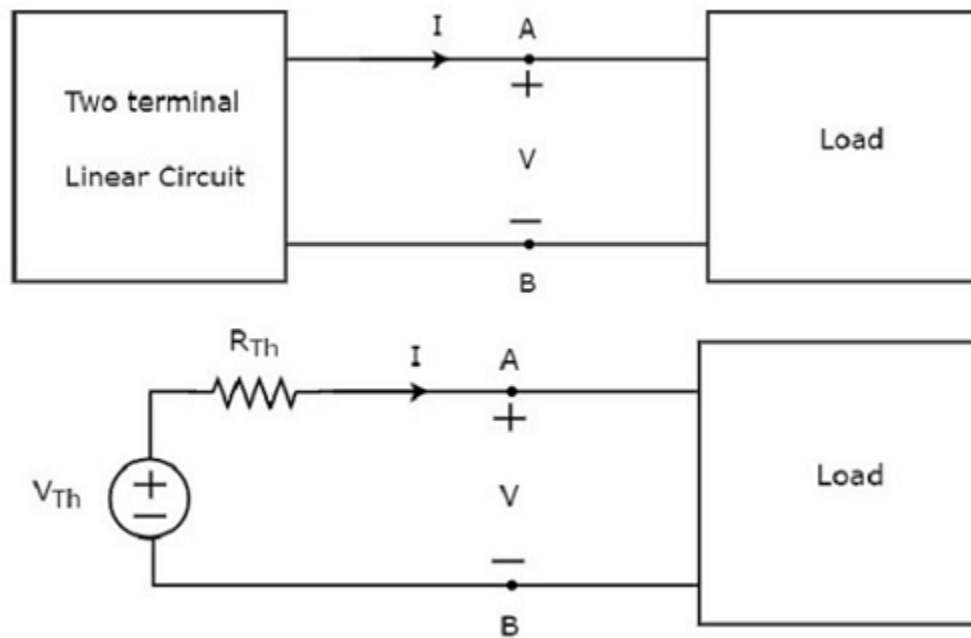
### Q.3 a) writes statement of Thevenin's theorems.

ANS: Thevenin's theorem states that any two terminal linear network or circuit can be represented with an equivalent network or circuit, which consists of a voltage source in series with a resistor. It is known as Thevenin's equivalent circuit. A linear circuit may contain independent sources, dependent sources, and resistors.

If the circuit contains multiple independent sources, dependent sources, and resistors, then the response in an element can be easily found by replacing the entire network to the left of that element with a Thevenin's equivalent circuit.

The response in an element can be the voltage across that element, current flowing through that element, or power dissipated across that element.

This concept is illustrated in following figures.



Thevenin's equivalent circuit resembles a practical voltage source. Hence, it has a voltage source in series with a resistor.

- The voltage source present in the Thevenin's equivalent circuit is called as Thevenin's equivalent voltage or simply Thevenin's voltage,  $V_{Th}$ .
- The resistor present in the Thevenin's equivalent circuit is called as Thevenin's equivalent resistor or simply Thevenin's resistor,  $R_{Th}$ .

### Methods of Finding Thevenin's Equivalent Circuit

There are three methods for finding a Thevenin's equivalent circuit. Based on the type of sources that are present in the network, we can choose one of these three methods. Now, let us discuss two methods one by one. We will discuss the third method in the next chapter.

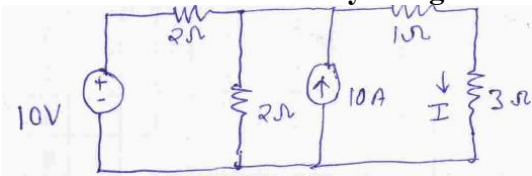
Follow these steps in order to find the Thevenin's equivalent circuit, when only the sources of independent type are present.

- Step 1 – Consider the circuit diagram by opening the terminals with respect to which the Thevenin's equivalent circuit is to be found.
- Step 2 – Find Thevenin's voltage  $V_{Th}$  across the open terminals of the above circuit.
- Step 3 – Find Thevenin's resistance  $R_{Th}$  across the open terminals of the above circuit by eliminating the independent sources present in it.
- Step 4 – Draw the Thevenin's equivalent circuit by connecting a Thevenin's voltage  $V_{Th}$  in series with a Thevenin's resistance  $R_{Th}$ .



Now, we can find the response in an element that lies to the right side of Thevenin's equivalent circuit.

b) Find the current in  $3\Omega$  resistor by using Thevenin's theorems.



ANS:

ANS. 3  
b).

$R_{th}$

$$R_{th} = (2 \parallel 2) + 1 = 2\Omega$$

$V_{th}$

$$V_x = V_{th}$$

$$\frac{V_x - 10}{2} + \frac{V_x}{2} - 10 + 0 = 0$$

$$V_x = 20V$$

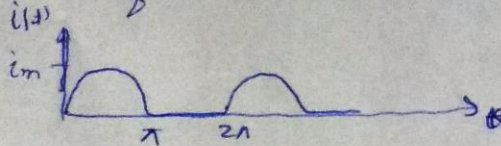
Current in  $3\Omega$  is

$$I_{3\Omega} = \frac{V_{th}}{R_{th} + R_L} = \frac{20}{3 + 2} = 4 \text{ Amps}$$

Q.4 find effective value, mean value, form factor and peak factor for half rectified voltage sinusoidal waveform.

ANS.

Output of HWR is



$$\text{the equation is } i = \begin{cases} i_m \sin \theta, & 0 \leq \theta \leq \pi \\ 0, & \pi \leq \theta \leq 2\pi \end{cases}$$

$$\textcircled{1} V_{avg} = \frac{1}{2\pi} \int_0^{2\pi} i d\theta = \frac{1}{2\pi} \left[ \int_0^{\pi} i d\theta + \int_{\pi}^{2\pi} i d\theta \right]$$

$$\Rightarrow \frac{1}{2\pi} \left[ \int_0^{\pi} i_m \sin \theta d\theta + 0 \right] = \frac{i_m}{2\pi} [-\cos \theta]_0^{\pi}$$

$$\left[ V_{avg} = -\frac{i_m}{2\pi} [-1-1] = \frac{i_m}{\pi} \right] - \underline{\underline{ANS}}$$

$\textcircled{2}$

~~V<sub>rms</sub>~~  
Rms  
value

$$I = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i^2 d\theta} = \sqrt{\frac{1}{2\pi} \left[ \int_0^{\pi} i^2 d\theta + \int_{\pi}^{2\pi} i^2 d\theta \right]}$$

$$I^2 = \sqrt{\frac{1}{2\pi} \left[ \int_0^{\pi} i_m^2 \sin^2 \theta d\theta + 0 \right]} = \sqrt{\frac{i_m^2}{2\pi} \cdot \frac{1}{2} \int_0^{\pi} (1 - \cos 2\theta) d\theta}$$

$$I = \sqrt{\frac{i_m^2}{4\pi} \cdot \pi} =$$

$$\boxed{I = \frac{i_m}{\sqrt{2}}} - \underline{\underline{ANS}}$$

$$\textcircled{3} K_F = \frac{I_{rms}}{I_{avg}} = \frac{i_m/2}{i_m/\pi} = \frac{\pi}{2} = 1.57 \quad \underline{\underline{ANS}}$$

$\textcircled{4}$

$$K_p = \frac{I_m}{I_{rms}} = \frac{i_m}{i_m/2} = 2 \quad \underline{\underline{ANS}}$$

Q.5 explains maximum power transfer theorems in details and calculates max. Power.

ANS: Maximum power transfer theorem states that the DC voltage source will deliver maximum power to the variable load resistor only when the load resistance is equal to the source resistance.

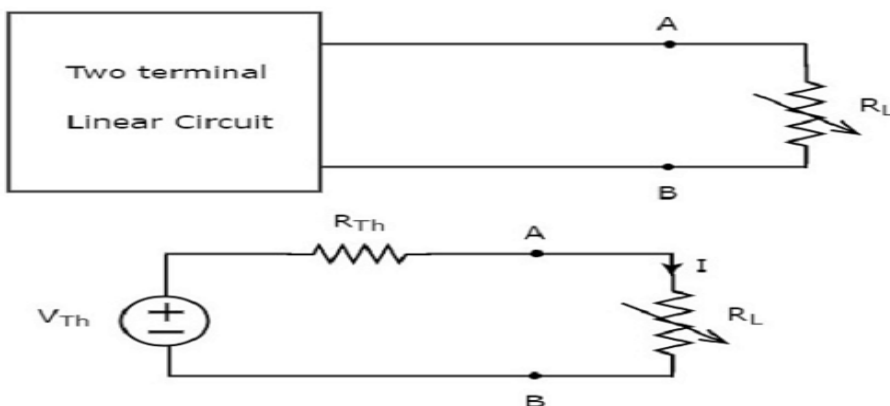
Similarly, Maximum power transfer theorem states that the AC voltage source will deliver maximum power to the variable complex load only when the load impedance is equal to the complex conjugate of source impedance.

In this chapter, let us discuss about the maximum power transfer theorem for DC circuits.

#### Proof of Maximum Power Transfer Theorem

Replace any two terminal linear network or circuit to the left side of variable load resistor having resistance of  $R_L$  ohms with a Thevenin's equivalent circuit. We know that Thevenin's equivalent circuit resembles a practical voltage source.

This concept is illustrated in following figures.



The amount of power dissipated across the load resistor is

$$P_L = I^2 R_L$$

Substitute  $I = \frac{V_{Th}}{R_{Th} + R_L}$  in the above equation.

$$P_L = \left[ \frac{V_{Th}}{R_{Th} + R_L} \right]^2 R_L$$

$$\Rightarrow P_L = \frac{V_{Th}^2 R_L}{(R_{Th} + R_L)^2}$$

#### Condition for Maximum Power Transfer

For maximum or minimum, first derivative will be zero. So, differentiate Equation 1 with respect to  $R_L$  and make it equal to zero.

$$\frac{dP_L}{dR_L} = \frac{V_{Th}^2}{(R_{Th} + R_L)^2} \times 1 - \frac{V_{Th}^2 R_L \times 2(R_{Th} + R_L)}{(R_{Th} + R_L)^4} = 0$$

$$\Rightarrow \frac{V_{Th}^2}{(R_{Th} + R_L)^2} - \frac{2V_{Th}^2 R_L}{(R_{Th} + R_L)^3} = 0 \Rightarrow \frac{1}{(R_{Th} + R_L)^2} - \frac{2R_L}{(R_{Th} + R_L)^3} = 0$$

$$\Rightarrow \frac{1}{(R_{Th} + R_L)^2} - \frac{2R_L}{(R_{Th} + R_L)^3} = 0 \Rightarrow \frac{1}{(R_{Th} + R_L)^2} - \frac{2R_L}{(R_{Th} + R_L)^3} = 0$$



$$\Rightarrow (R_{Th} - R_L) = 0 \Rightarrow (R_{Th} - R_L) = 0$$

$$\Rightarrow R_{Th} = R_L \text{ or } R_L = R_{Th} \Rightarrow R_{Th} = R_L \text{ or } R_L = R_{Th}$$

Therefore, the condition for maximum power dissipation across the load is  $R_L = R_{Th}$ . That means, if the value of load resistance is equal to the value of source resistance i.e., Thevenin's resistance, then the power dissipated across the load will be of maximum value.

The value of Maximum Power Transfer

Substitute  $R_L = R_{Th}$  &  $P_L = P_{L,Max}$  in Equation 1.

$$P_{L,Max} = V_{Th}^2 \{ R_{Th} / (R_{Th} + R_{Th})^2 \} \Rightarrow P_{L,Max} = V_{Th}^2 \{ R_{Th} / (2R_{Th})^2 \}$$

$$P_{L,Max} = V_{Th}^2 \{ R_{Th} / 4R_{Th}^2 \} \Rightarrow P_{L,Max} = V_{Th}^2 \{ 1 / 4R_{Th} \}$$

$$\Rightarrow P_{L,Max} = V_{Th}^2 / 4R_{Th} \Rightarrow P_{L,Max} = V_{Th}^2 / 4R_{Th}$$

$$\Rightarrow P_{L,Max} = V_{Th}^2 / 4R_L, \text{ since } R_L = R_{Th} \Rightarrow P_{L,Max} = V_{Th}^2 / 4R_L, \text{ since } R_L = R_{Th}$$

Therefore, the maximum amount of power transferred to the load is

$$P_{L,Max} = V_{Th}^2 / 4R_L = V_{Th}^2 / 4R_{Th} \Rightarrow P_{L,Max} = V_{Th}^2 / 4R_L = V_{Th}^2 / 4R_{Th}$$

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**JAGANNATH GUPTA INSTITUTE OF ENGINEERING & TECHNOLOGY JAIPUR**  
I / II - MID TERM PAPER ANSWER SHEET

Semester: II  
Subject: EM

Branch: All Branch  
Submitted by: Ms. Daljit Kaur

- (1) Two forces acting at a point may be as shown in fig.

where  $F_1 = 100 \text{ KN}$

$F_2 = 50 \text{ KN}$ ,  $\theta = 60^\circ$

Using Isgm Law of forces

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta}$$

$$= \sqrt{100^2 + 50^2 + 2 \times 100 \times 50 \cos 60^\circ}$$

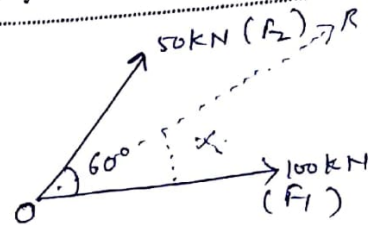
$$= \sqrt{132.3} \text{ KN}$$

The direction of  $R$  is  $\tan \alpha = \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta}$   
where  $\alpha$  is the angle between  $F_1$  and  $R$

Thus  $\tan \alpha = \frac{50 \sin 60^\circ}{100 + 50 \cos 60^\circ} = \frac{\sqrt{3}}{5}$

$$\alpha = \tan^{-1} \left( \frac{\sqrt{3}}{5} \right) = 19.1^\circ$$

The angle with  $F_2$  is  $(60^\circ - 19.1^\circ) = 40.9^\circ$ .

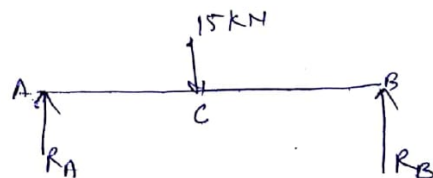
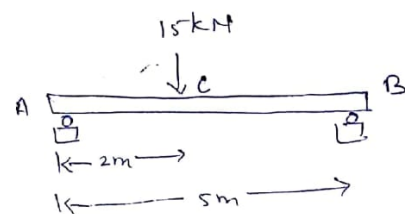


- (2) As per problem statement the configuration is as shown in fig.

Beam is in equilibrium under the following forces

- (1) 15 KN acting at C
- (2) Reaction force  $R_A$  acting at A
- (3) Reaction force  $R_B$  acting at B

Assuming the virtual displacement given in vertical upper direction at point B is  $y$ .



Total work done by these forces due to virtual work must be zero.

Thus

$$0 \times R_A - CC' \times 15 + BB' \times R_B = 0$$

$$R_B = 15 \frac{CC'}{BB'}$$

From  $\triangle ACC'$  and  $\triangle ABB'$

$$\frac{AC}{AB} = \frac{CC'}{BB'} \Rightarrow \frac{2}{5} = \frac{CC'}{BB'}$$

$$\therefore R_B = 15 \cdot \frac{2}{5} = 6 \text{ KN}$$

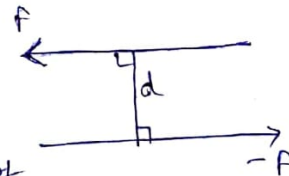
Now resolving forces vertically, we get

$$R_A + R_B = 15$$

$$R_A = 15 - 6 = 9 \text{ KN}$$

- (3) Couple When two parallel forces that have the same magnitude but opposite direction is known as couple. The couple is separated by perpendicular distance. As matter of fact a couple is unable to produce any straight line motion but it produces rotation in the body on which it acts. So, couple can be defined as unlike parallel forces of same magnitude but opposite direction which produce rotation about a specific direction & whose resultant is zero

The perpendicular distance between the lines of action of the constituent forces is called an arm of the couple. In fig! the constituent forces being  $F$  and  $-F$ , the arm of the couple is  $d$ .



Clockwise couple:- A couple whose tendency is to rotate the body in a clockwise direction is known as clockwise couple.



Anticlockwise couple :- A couple whose tendency is to rotate the body in anticlockwise direction is known as anticlockwise couple.

- ④ A particle is in equilibrium if it is at rest if originally at rest or has a constant velocity if originally in motion. The term equilibrium or static equilibrium is used to describe an object at rest. To maintain equilibrium it is necessary to satisfy Newton's first law of motion, which requires the resultant force acting on particle to be equal to zero.

i.e.  $\Sigma F = 0$  where  $\Sigma F$  = sum of all the forces acting on the particle.

Conditions of Equilibrium :- Consider a body acted upon by a number of coplanar non-concurrent forces. As a result of these forces, the body may have one of the following states:

1. The body may move in any one direction
2. The body may rotate about itself without moving
3. The body may move in any one direction & at the same time it may also rotate about itself
4. The body may be completely at rest.

Condition 1: If the body moves in any direction, it means that there is a resultant force acting on it. A little consideration will show, that if the body is to be at rest or in equilibrium, the resultant force causing movement must be zero. Or in other words, the horizontal component of all the forces ( $\Sigma H$ ) & vertical component of all the forces ( $\Sigma V$ ) must be zero.

Mathematically

$$\Sigma H = 0 \text{ \& \> } \Sigma V = 0$$

2. If the body rotates about itself, without moving, it means that there is a single resultant couple acting on it with no resultant force. A little consideration will show, if the body is to be at rest or in equilibrium, the moment of the couple causing rotation must be zero. i.e. the resultant moment of all the forces ( $\Sigma M$ ) must be zero.

Mathematically

$$\Sigma M = 0$$

3. If the body moves in any direction & at the same time it rotates about itself, it means that there is a resultant force & also a resultant couple acting on it. A little

Consideration will show, that if the body is to be at rest or in equilibrium, the resultant force causing movement & the resultant moment of the couple causing rotation must be zero. i.e. horizontal component of all the forces ( $\Sigma H$ ), vertical component of all the forces ( $\Sigma V$ ) & the resultant moment of all the forces ( $\Sigma M$ ) must be zero.

Mathematically,

$$\Sigma H = 0 \quad \Sigma V = 0 \quad \& \quad \Sigma M = 0$$

4. If the body is completely at rest, it necessarily means that there is neither a resultant force nor a couple acting ~~at~~ on it. In this case the following conditions are already satisfied:

$$\Sigma H = 0 \quad \Sigma V = 0 \quad \& \quad \Sigma M = 0$$

These three equations are known as the conditions of equilibrium.

⑤ Let force 100 N is applied at distance from point B. All acting forces on ladder is shown in fig. Consider the FBD of ladder.

Applying the condition of equilibrium

$$\Sigma F_x = 0 \quad R_A \cos 60^\circ - R_B \cos 45^\circ = 0$$

$$R_A = 1.414 R_B$$

$$\Sigma F_y = 0 \quad R_A \sin 60^\circ + R_B \sin 45^\circ - 200 - 100 = 0$$

Substituting  $R_A = 1.414 R_B$  in

$$1.414 R_B \sin 60^\circ + R_B \sin 45^\circ = 300$$

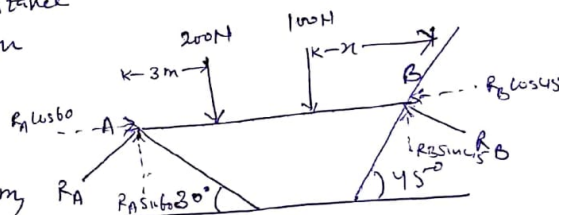
$$R_B = 155.3 \text{ N}$$

$$R_A = 1.414 (155.3) = 219.6 \text{ N}$$

$\Sigma M_B = 0$  {moment equilibrium about point B}

$$(-R_A \sin 60^\circ \times 12) + (200 \times 9) + 100x = 0$$

$$\text{Thus, } x = 4.82 \text{ m}$$



---

**Note: Attempt any four questions. Every question carries equal marks. Question No. 1 is compulsory to attempt.**

1. Write short notes on any four of followings:-
  - a. Common impurities of water
  - b. Degree of hardness
  - c. Requisites of drinking water
  - d. Diagram of 'Zeolite Method'
  - e. Break point Chlorination
  - f. Diagram of 'Cold Lime- Soda Method'
2. What is Hardness? Explain EDTA method to determine hardness of water.
3. Calculate the amount of Lime and Soda required for softening 1,00,000 liters of water containing  $\text{Ca}(\text{HCO}_3)_2 = 8.1 \text{ mg/L}$ ,  $\text{Mg}(\text{HCO}_3)_2 = 7.5 \text{ mg/L}$ ,  $\text{CaSO}_4 = 13.6 \text{ mg/L}$ ,  $\text{MgSO}_4 = 12.0 \text{ mg/L}$  and  $\text{MgCl}_2 = 2.0 \text{ mg/L}$  and  $\text{NaCl} = 4.7 \text{ mg/L}$ . Purity of lime is 90% and that of soda is 98%.
4. Explain municipal water supply including sedimentation, filtration and disinfection.
5. Explain hot lime soda method of water softening including chemical reaction.

**OR**

Explain scale and sludge formation in the boiler and their removal methods.



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**JAGANNATH GUPTA INSTITUTE OF ENGINEERING & TECHNOLOGY JAIPUR**  
I / II - MID TERM PAPER ANSWER SHEET

Semester: 2<sup>nd</sup>

Branch: All

Subject: Engineering Chemistry

Submitted by: Dr. Pankaj Sharma

Ans. (1); (a) water is a great solvent & it dissolves and sustain numerous types of impurities. common impurities present in natural water may be classified as follows: (A) Suspended Impurities:

- (i) Inorganic Impurities, e.g. sand & soil
- (ii) organic Impurities, e.g. vegetables, oil, organic matter
- (iii) colloidal Impurities, e.g. colouring matter, finely divided silica, clay etc.
- (iv) Micro-Organisms, e.g. Bacteria, fungi, algae

(B) Dissolved Impurities: (i) Inorganic Impurities like carbonates, bicarbonates, Nitrates, sulphates, chlorides of Ca, Mg, Fe, Na, K, Al, Cu & Zn etc.

(ii) Dissolved Gases like  $\text{CO}_2$ ,  $\text{SO}_2$ ,  $\text{N}_2$ ,  $\text{NH}_3$  etc.

(iii) Dissolved organic Matter: Domestic sewage and Industrial waste contain organic compounds like Urea, phenols, Amines, Aldehydes, pesticides etc.

(C) Biological Impurities: various pathogenic micro-organisms like bacteria, viruses, fungi, algae etc. Helminths, Protozoas etc.

Ans. (1)(b) Hardness of water depends upon the amount of Calcium, Magnesium & Heavy metals salts present in it. The degree of hardness of water is usually expressed as equivalent amount of  $\text{CaCO}_3$ . This is because of two reasons:

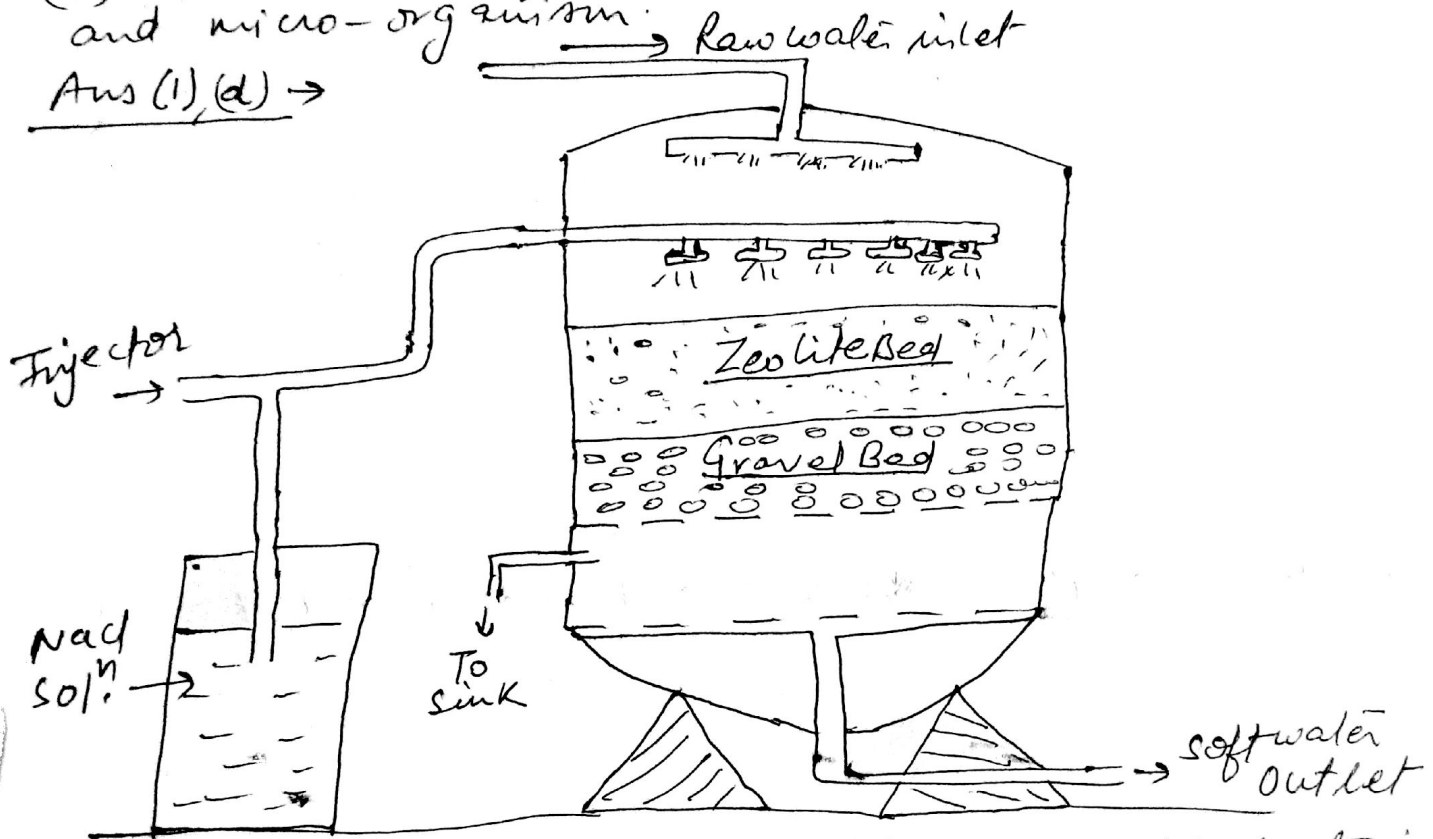
- ① Its molecular weight is 100 (Ease for calculation)
- ② It is the most insoluble salt that can be precipitated in water treatment.

$$\text{CaCO}_3 \text{ Equivalent of Hardness producing substance} = \frac{\text{Weight of Hardness producing substance}}{\text{Chemical equivalent of CaCO}_3 (\text{i.e. 50})} \times \frac{\text{Chemical equivalent of Hardness producing substance}}{\text{Chemical equivalent of CaCO}_3 (\text{i.e. 50})}$$

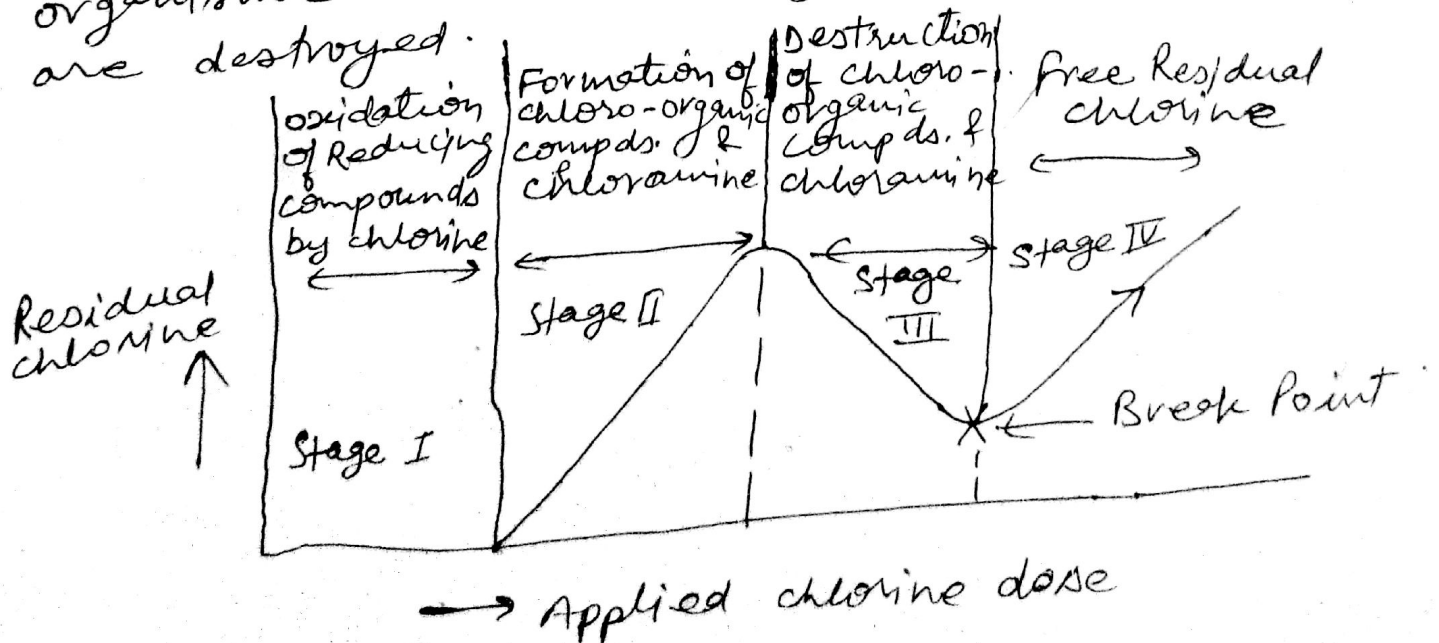
Page 2  
Ans (1), (c) → (i) ~~the~~ water should be colourless, odourless and pleasant in taste.

- (ii) pH should be in the range 6.5 - 8.0
- (iii) Total dissolved solid / Hardness should be less than 500 ppm.
- (iv) chlorides and sulphates must be less than 250 ppm.
- (v) Turbidity should be less than 10 ppm.
- (vi) Fluoride should be less than 1.5 ppm.
- (vii) It should be free from Lead, Arsenic, Mercury & Chromium.
- (viii) It should be free from Harmful gases like  $H_2S$ .
- (ix) It should be free from disease producing bacteria and micro-organism.

Ans (1), (d) →



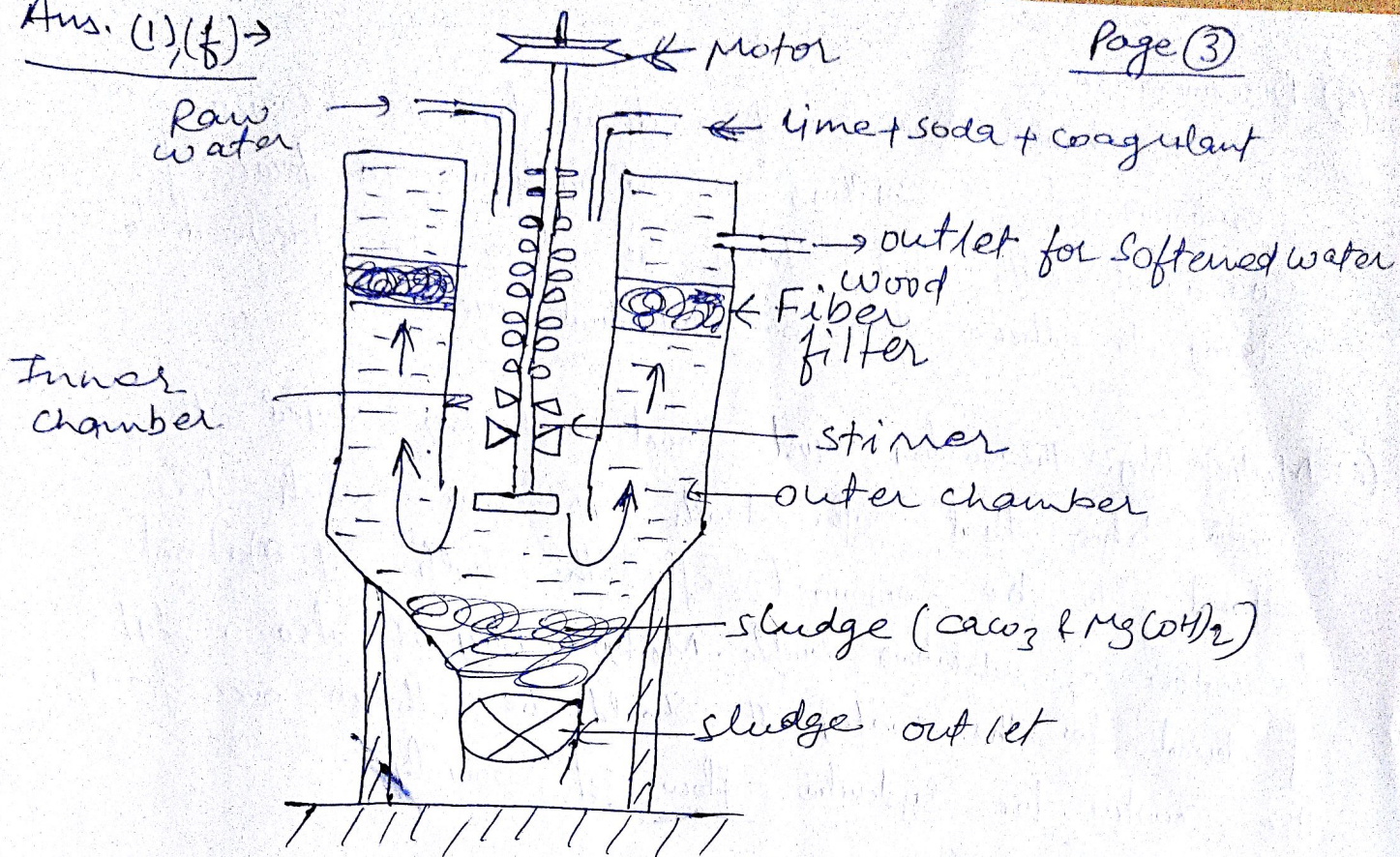
Ans (1), (e) → Break Point chlorination is the chlorination of water to such an extent that not only living organism but also other organic impurities in water are destroyed.



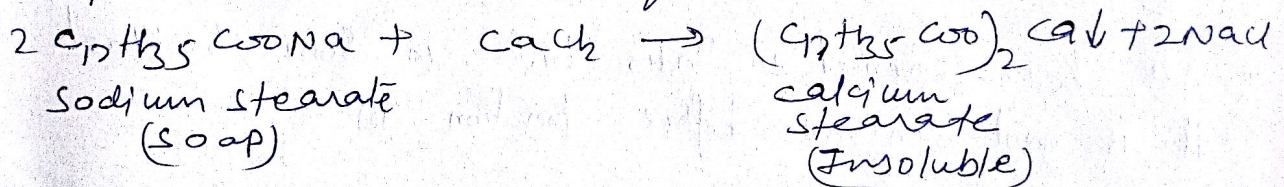


Ans. (1)(f) →

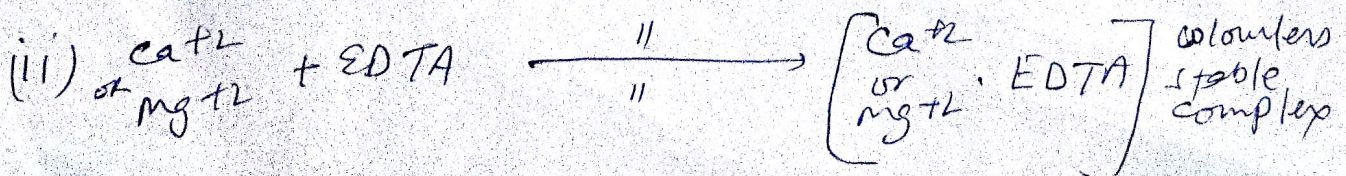
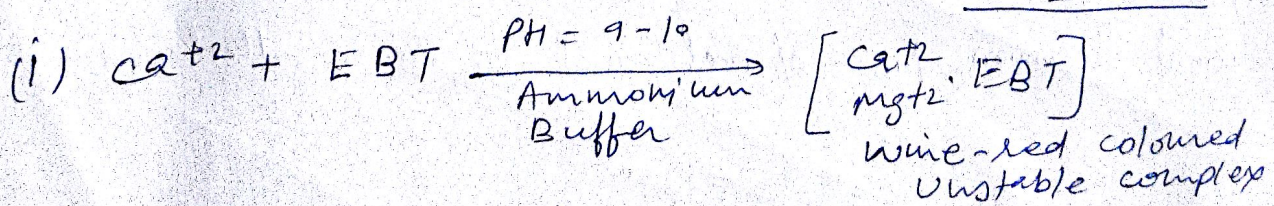
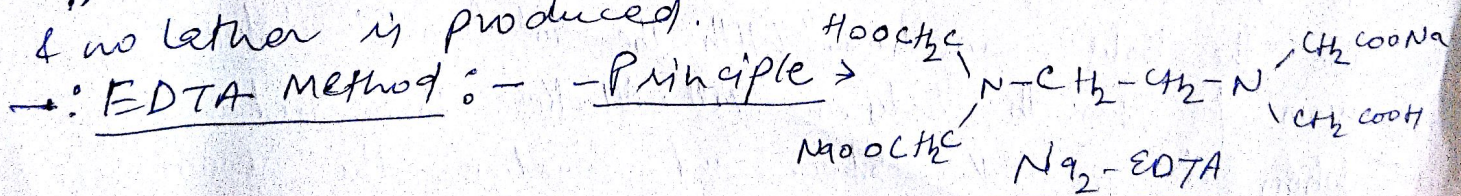
Page (3)



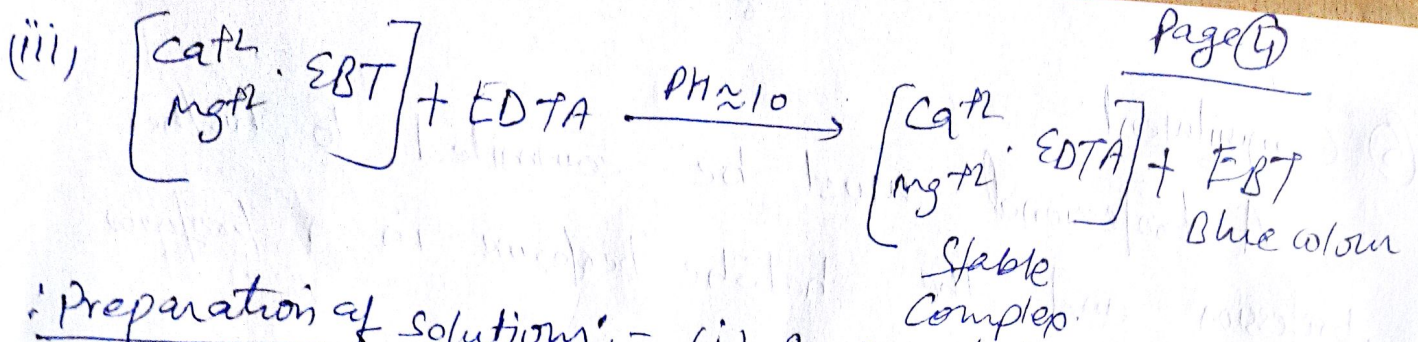
Ans (2) → water which does not produce lather with soap solution rapidly is called 'Hard water', water which produce lather easily with soap solution is called soft water. Thus, Hardness is the characteristic of water 'which prevents Lathering of soap'.



Hardness produced by salts of calcium & magnesium reacts with soap & produce insoluble precipitate & no lather is produced.







Preparation of solutions:- (i) Preparation of standard Hardwater → Dissolve 1.0 gm of pure dry  $\text{CaCO}_3$  in minimum quantity of dil. HCl & soln is evaporated to dryness. Dissolve the residue in distilled water & make up the soln one litre. Thus 1 ml. of this soln contains 1 mg  $\text{CaCO}_3$  hardness.

(ii) Preparation of EDTA soln → Dissolve 3.72 gm Disodium salt of EDTA (Mol. Mass = 372.25 gm), in distilled water & make up to 1 litre to get M/100 EDTA soln.

(iii) Preparation of Indicator solution → 0.5 gm of Eriochrome Black-T is dissolved in 100 ml alcohol.

(iv) Preparation of Buffer (pH ≈ 10) → Dissolve 67.5 gm of NaHCl in 570 ml of Ammonia & make up to 1 litre with distilled water.

— Procedure → Step I: Standardisation of EDTA soln →

50 ml. standard H. water is taken in a conical flask add 8-10 ml. Buffer soln & 2-3 drops of EBT Indicator. Titrate the above soln with EDTA soln from burette to get end point (wine red colour changes to Blue).

Let the vol. of EDTA consumed ' $V_1$ ' ml.

Step II: Determination of Total Hardness of Sample water  
take 50 ml. of H. w. sample in conical flask & follow the same procedure as mentioned above.

Let the vol. of EDTA consumed be ' $V_2$ ' ml.

Step III: Estimation of Permanent Hardness →

take 250 ml. Hard water sample in a beaker & boil it till vol. reduces half. On Boiling the temp. Hardness will be removed. The soln is cooled, filtered and filtrate is made up to 250 ml with distilled water. 50 ml. of this soln is taken & the same procedure is repeated as above step.

Let the vol. of EDTA consumed be ' $V_3$ ' ml.

P.T.O.



Calculation : (I) 50 ml. of standard Hard water =  $V_1$  ml. of EDTA sol<sup>n</sup>?  
 $50 \times 1 \text{ mg of CaCO}_3 \text{ eq.} = V_1 \text{ ml. of EDTA sol}^n$   
 $\therefore 1 \text{ ml. of EDTA sol}^n = \frac{50}{V_1} \text{ mg of CaCO}_3 \text{ eq.}$

(II) Total Hardness :

50 ml. Sample H. water =  $V_2$  ml of EDTA sol<sup>n</sup>?  
 $\therefore 1 \text{ ml} = V_2 \times \frac{50}{V_1} \text{ mg of CaCO}_3 \text{ eq.}$

$\therefore 1000 \text{ ml} = \frac{V_2}{50} \times \frac{50}{V_1} \times 1000 \text{ mg/L OR ppm as CaCO}_3 \text{ eq.}$

(III) Permanent Hardness :

50 ml. Boiled water sample =  $V_3$  ml. EDTA sol<sup>n</sup>?  
 $= V_3 \times \frac{50}{V_1} \text{ mg of CaCO}_3 \text{ eq.}$

$\therefore 1 \text{ ml} =$

$\therefore 1000 \text{ ml} = \frac{V_3}{50} \times \frac{50}{V_1} \times 1000 \text{ mg/L OR ppm as CaCO}_3 \text{ eq.}$

(IV) Temporary Hardness  $\rightarrow$

Temp. Hardness = Total Hardness - Perm. Hardness

$$= \left( \frac{V_2}{V_1} \times 1000 - \frac{V_3}{V_1} \times 1000 \right) \text{ mg/L OR ppm as CaCO}_3 \text{ eq.}$$

$$= \left( \frac{V_2 - V_3}{V_1} \right) \times 1000 \text{ mg/L OR ppm as CaCO}_3 \text{ eq.}$$

Page 6

Ans. (3) →	CaCO <sub>3</sub> equivalent	Requirement
$\text{Ca(HCO}_3)_2 = 8.1 \text{ mg/L}$	$8.1 \times \frac{50}{81} = 5.0 \text{ mg/L}$	L
$\text{Mg(HCO}_3)_2 = 7.5 \text{ mg/L}$	$7.5 \times \frac{50}{73} = 5.14 \text{ mg/L}$	2L
$\text{CaSO}_4 = 13.6 \text{ mg/L}$	$13.6 \times \frac{50}{68} = 5.0 \text{ mg/L}$	S
$\text{MgSO}_4 = 12.0 \text{ mg/L}$	$12 \times \frac{50}{60} = 10.0 \text{ mg/L}$	L + S
$\text{MgCl}_2 = 2.0 \text{ mg/L}$	$2 \times \frac{50}{47.5} = 2.11 \text{ mg/L}$	L + S
$\text{NaCl} = 4.7 \text{ mg/L}$	No cal. required.	—

lime required for 1,00,000 litres water; when lime is 90% Pure,

$$= \frac{74}{100} \left[ \text{Ca(HCO}_3)_2 + 2 \times \text{Mg(HCO}_3)_2 + \text{MgSO}_4 + \text{MgCl}_2 \right] \times \frac{100}{90}$$

mg/L as CaCO<sub>3</sub> eq.

$$= \frac{74}{100} [5 + 2 \times 5.14 + 10 + 2.11] \times \frac{1,00,000 \times 100}{106 \times 90} \text{ kg}$$

$$= 2.0268 \times \frac{100}{98} \text{ kg}$$

$$= 2.252 \text{ kg lime}$$

Soda required for 1,00,000 L water, when Soda is 98% Pure,

$$= \frac{106}{100} \left[ \text{CaSO}_4 + \text{MgSO}_4 + \text{MgCl}_2 \right] \times \frac{100}{98} \times \frac{100,000}{106} \text{ kg}$$

mg/L as CaCO<sub>3</sub> eq.

$$= \frac{106}{100} [5 + 10 + 2.11] \times \frac{100}{98} \times \frac{1}{10} \text{ kg}$$

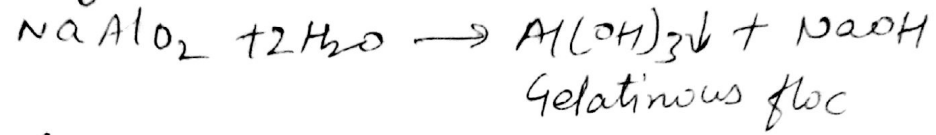
$$= 1.812 \times \frac{100}{98} = 1.848 \text{ kg soda}$$

Ans. 4 → Municipal water treatment is a process to supply safe drinking water to <sup>Human</sup> population. It includes

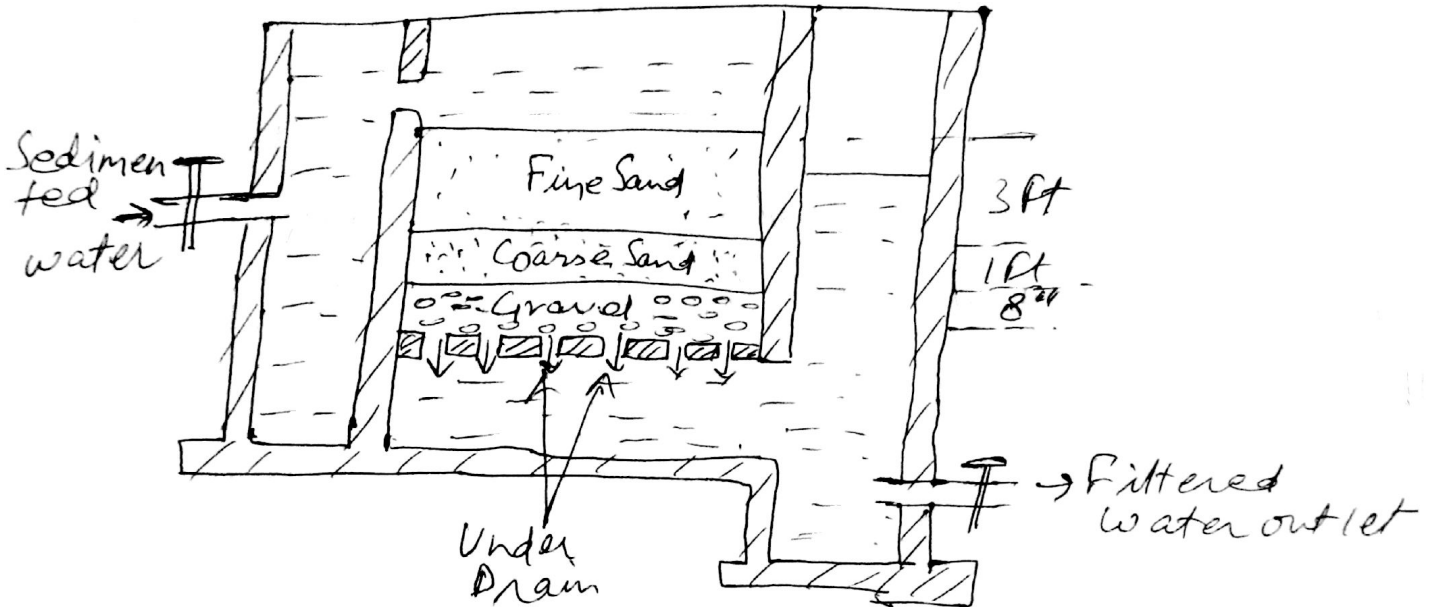
(a) screening → Raw water is passed through screens having large number of holes which retain floating materials.

(b) Sedimentation → It is a process of allowing water to stand undisturbed in big tanks for 2-6 hours. Most of suspended particles settle down at the bottom due to force of gravity. The clear supernatant is drawn from tank with the help of pumps.

If water contain colloidal and fine clay particles, we apply Sedimentation with coagulation. Coagulants such as Alum and Sodium Aluminate produce gel ppt of  $Al(OH)_3$  and Ferrous sulphate produce gel ppt of  $Fe(OH)_3$ , which neutralise (+)ve charge on the colloidal particles, thus fine particles come close & combine to form bigger particles, which settle down due to gravity.



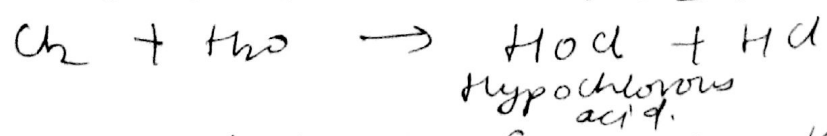
(c) Filtration → In this process we use sand filters to remove colloidal matter, Bacteria, microorganisms by passing water through a bed of fine sand and other proper sized granular materials.



Sand Filter

(d) Disinfection: (i) By Boiling → We Boil water for 15-20 minutes, all the disease producing bacteria are killed, water become safe for use.

(ii) By adding Bleaching Powder → 1kg of Bleaching Powder per 1000 Kilolitres of water is mixed and water allowed to stand undisturbed for several hours.



Germ + HOCl → Germ are Killed  
Germicidal  
in Action



Disadvantages: ① It introduces calcium

Hardness in water. ② It deteriorates on storage

③ Excess amount of B.P. produce bad taste & odour.

(ii) Chlorination Method → chlorine gas or concentrated water produces hypochlorous acid in water, which is a powerful germicide.



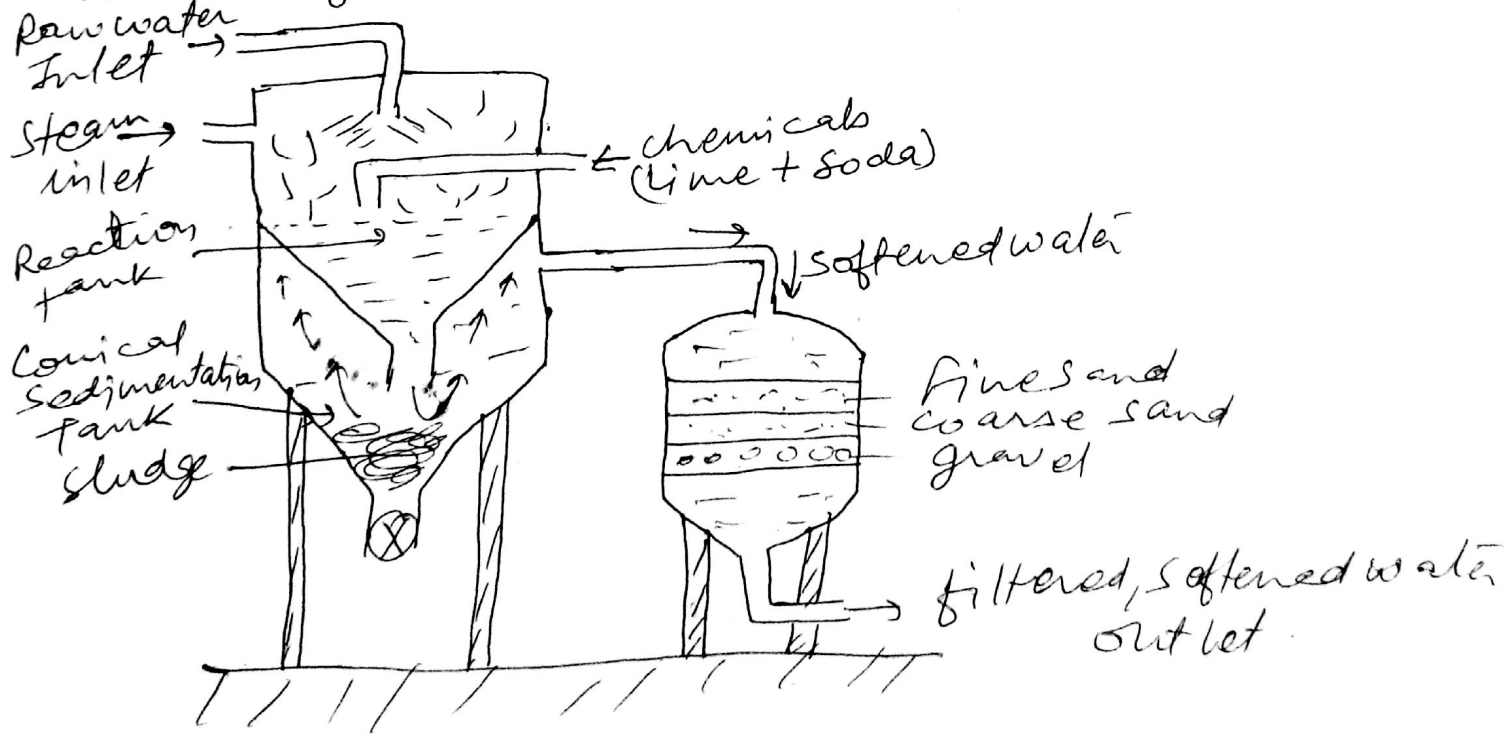
$\text{HOCl} + \text{Germs} \rightarrow$  Bacteria are killed.

$\text{HOCl}$ , inactivate the enzymes of Bacteria cell thus bacteria are killed. Filtered water requires about 0.3 to 0.5 ppm of chlorine

Advantages → ① effective and economical ② It is stable and does not deteriorate ③ It does not impart Hardness

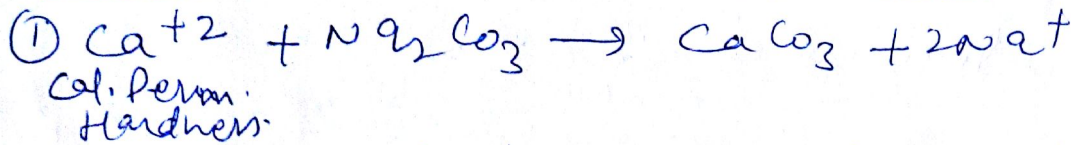
Disadvantages → ① Excess of chlorine, if added, produces a characteristic unpleasant taste & odour. The quantity of free chlorine in treated water should not exceed 0.1 to 0.2 ppm.

Ans 5 → Hot lime - Soda Method → we treat water with lime-soda at a temperature of  $30^\circ\text{C}$  to  $150^\circ\text{C}$ . The reaction proceed faster, precipitate and sludge settle down easily and rapidly. This method produces water of residual hardness of 15 to 30 ppm.

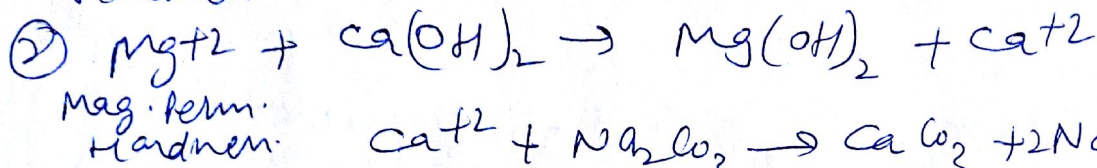


Chemical reactions of L-S method →

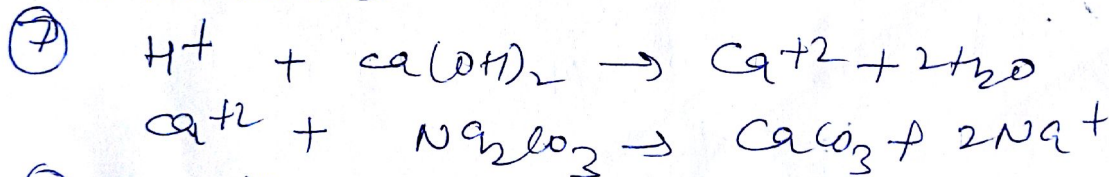
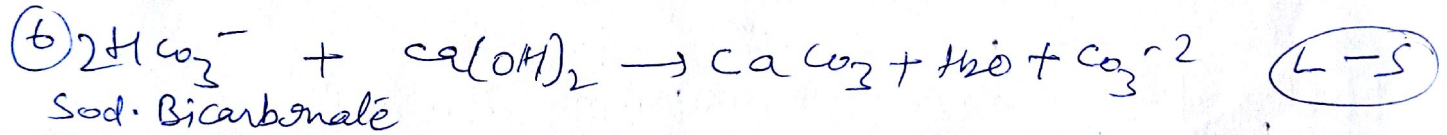
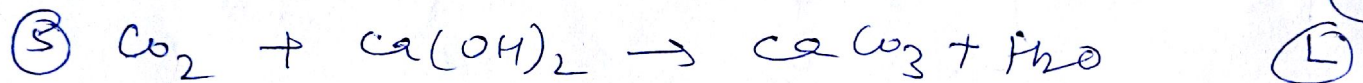
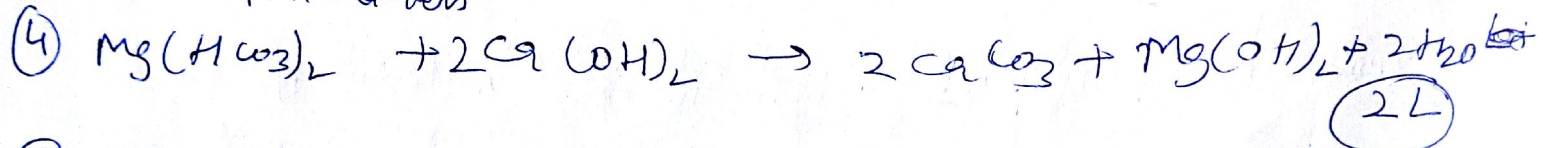
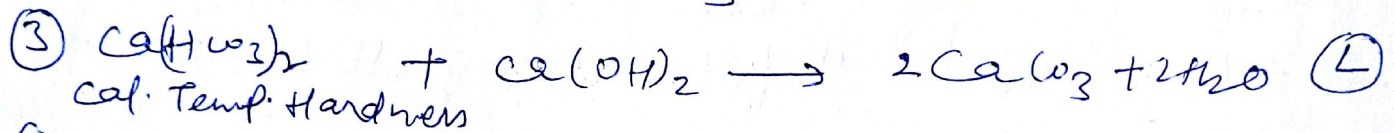
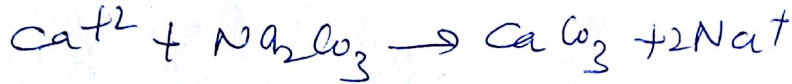
Requirement



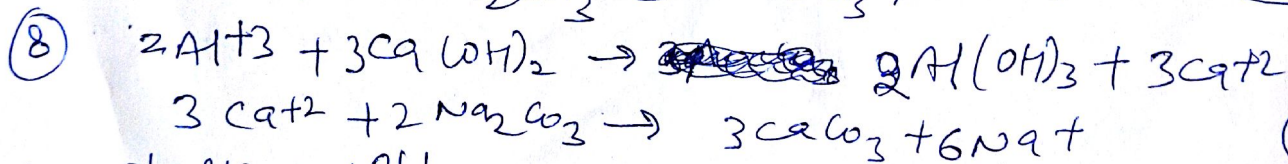
(S)



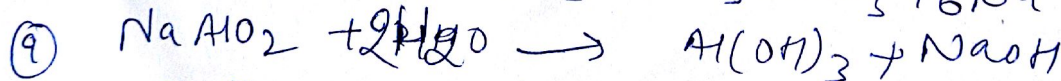
(L+S)



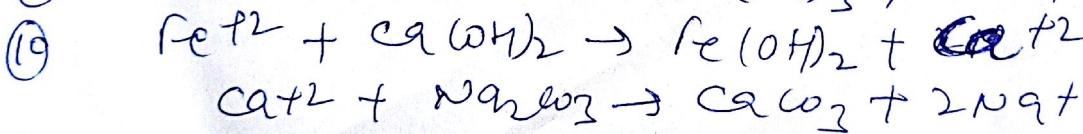
(L+S)



(L+S)



(-L)



(L+S)

## JAGANNATH GUPTA INSTITUTE OF ENGINEERING &amp; TECHNOLOGY, JAIPUR

## I-MID TERM PAPER SOLUTIONS

SEMESTER : II-SEM

BRANCH : ALL BRANCHES

SUBJECT : HUMAN VALUES

SUBMITTED BY : JYOTSNA SAINI

**Note: Attempt any four questions out of five questions.**

- Q1. What do you understand by the term **“HUMAN VALUES”**? How it help us in value Education explain ?
- Q2. Explain the term **“HARMONY”**. Brief the harmony in context to **“ MYSELF”**.
- Q3. Trace out the Difference between **“Sukh - Suvidha”** and **“Sanyam and Swasthaya”**
- Q4. Explain the term **“VISHWAS (Trust) and SAMMAN (Respect)”**
- Q5. Define the Term:-
- Intension
  - Competence

**Solutions**

Ans-1 Environmental ethics helps define man's moral and ethical obligations toward the environment. But **human values** become a factor when looking at environmental ethics. Human values are the things that are important to individuals that they then use to evaluate actions or events. In other words, humans assign value to certain things and then use this assigned value to make decisions about whether something is right or wrong. Human values are unique to each individual because not everyone places the same importance on each element of life. For example, a person living in poverty in an undeveloped country may find it morally acceptable to cut down the forest to make room for a farm where he can grow food for his family. However, a person in a developed country may find this action morally unacceptable because the destruction of forests increases carbon dioxide emissions into the atmosphere, which can negatively impact the environment.

Ans-2 We spend most of our time ‘with’ ourselves, ‘in’ ourselves but we spend most of our time thinking about other things and neglect ourselves.

The Self (I) is the basis of everything we do. All our desires, expectations, thoughts, feelings, understanding etc. come from the “I”.

Studying our “Self” helps us to have more clarity about ourselves and makes us confident. It develops our understanding and helps in building good relationships with everyone. It also helps us to understand our program better.

Ans-3 Sukh is a holistic and all encompassing state of the mind that creates inner harmony. Sukh is also called as happiness. Suvidha implies that it is looking for physical comforts and all the sources of attaining such comforts. When our body gets used to a certain level of comfort then we will only feel comfortable at that level e.g. comfort in fan, cooler or air conditioner. Different people have a different perception of suvidha and will seek a corresponding level of suvidha according to their perceptions.

By nature man is fond of comfort and happiness so he goes on making desires and ambitions one after the other to enjoy more in life. To lead a comfortable life he also accumulates many facilities, so that his life may become full of comfort and happiness. Sukh depends upon our thinking, so many times we are surrounded by materialistic possessions but we feel unsatisfied. People think that their happiness depends upon suvidha (facilities) but is it not so; happiness depends upon our thinking or our mental satisfaction.

Ans -4 normally we tend to believe that the body is an instrument for sensory enjoyment, which is not correct. We also happen to our body to exploit other human beings or rest of the nature, which is also not right utilization. Body is the instrument of the self and the body needs to be given nutrition, protection and utilized to work as an efficient and effective tool for the right purpose. This utilization is termed as right utilization. In other words, employing our body as an instrument for sensory enjoyment, and to exploit other human beings or rest of the nature is not the right utilization. On the contrary utilizing our body for right behavior and work is actually the right utilization of the body.

Ans:-5 An intention is an idea or plan of what you are going to do.

Competence: - A cluster of related abilities, commitments, knowledge, and skills that enable a person (or an organization) to act effectively in a job or situation.

Competence indicates sufficiency of knowledge and skills that enable someone to act in a wide variety of situations. Because each level of responsibility has its own requirements, competence can occur in any period of a person's life or at any stage of his or her career.

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JNIT

JAGANNATH GUPTA INSTITUTE OF ENGINEERING & TECHNOLOGY JAIPUR  
 ✓/ II - MID TERM PAPER ANSWER SHEET

Semester: II

Branch: ALL BRANCH

Subject: Maths - II

Submitted by: CHANDRESH MATHUR

Q.1 Test the consistency of the following system of Equation if possible  
 Solve them  
 $x + y + z = 6$  ;  $x - y + 2z = 5$  ,  $3x + y + z = 8$  ;  $2x - 2y + 3z = 7$

Ans The Given system of Linear Equation can be written as in Matrix form

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & 1 & 1 \\ 2 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 8 \\ 7 \end{bmatrix} \quad \text{i.e. } AX = B.$$

Augmented Matrix

$$C = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \\ 2 & -2 & 3 & 7 \end{bmatrix}$$

Applying the row operations.

$$R_2 \rightarrow R_2 - R_1 ; R_3 \rightarrow R_3 - 3R_1, R_4 \rightarrow R_4 - 2R_1$$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & 1 & -1 \\ 0 & -2 & -2 & -10 \\ 0 & -4 & 1 & -5 \end{bmatrix} \xrightarrow[R_4 \rightarrow R_4 - 2R_2]{R_3 \rightarrow R_3 - R_2} \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & 1 & -1 \\ 0 & 0 & -3 & -9 \\ 0 & 0 & -1 & -3 \end{bmatrix}$$

Applying  $R_4 \rightarrow R_4 - \frac{1}{3}R_3$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & 1 & -1 \\ 0 & 0 & -3 & -9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of C = Rank of A = No. of variable = 3.

System is consistent and have unique solution

$$\begin{aligned} x + y + z &= 6 \\ -2y + z &= -1 \\ 3z &= 9 \end{aligned}$$

$$x = 1, y = 2, z = 3.$$

Q.2 Find the Eigen value and Eigen Vector of the Matrix.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -6 \\ 2 & -2 & 3 \end{bmatrix}$$

Ans The Characteristic Equation of the Matrix.

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 2 & 0 \\ 2 & 1-\lambda & -6 \\ 2 & -2 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)[(1-\lambda)(3-\lambda)-12] - 2[2(3-\lambda)+12] = 0$$

$$\Rightarrow -\lambda^3 + 5\lambda^2 + 9\lambda - 45 = 0$$

$$(\lambda-3)(\lambda^2-2\lambda-15) = 0$$

$$(\lambda-3)(\lambda+3)(\lambda-5) = 0$$

$\lambda = 3, -3, 5$  are the Eigen values.

(i) Eigen vector corresponding to  $\lambda = 3$

$$\begin{bmatrix} -2 & 2 & 0 \\ 2 & -2 & -6 \\ 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Applying  $R_2 \rightarrow R_2 + R_1, R_3 \rightarrow R_3 + R_1$

$$\begin{bmatrix} -2 & 2 & 0 \\ 0 & 0 & -6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x_1 + 2y_1 = 0$$

$$-6z_1 = 0$$

$$\Rightarrow x_1 = y_1 = k_1 \text{ (let)}$$

$$z_1 = 0$$

if  $k=1$  then for  $\lambda=3$

the Eigen value  $x_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

(ii) For  $\lambda = -3$ . the Eigen vector  $x_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$

$$\begin{bmatrix} 4 & 2 & 0 \\ 2 & 4 & -6 \\ 2 & -2 & 6 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$



Applying  $R_2 \rightarrow 2R_2 - R_1$ ,  $R_3 \rightarrow 2R_3 - R_1$

$$\begin{bmatrix} 4 & 2 & 0 \\ 0 & 6 & -12 \\ 0 & -6 & 12 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{R_3 \rightarrow R_3 + R_2} \begin{bmatrix} 4 & 2 & 0 \\ 0 & 6 & -12 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$4x_2 + 2y_2 = 0$$

$$6y_2 - 12z_2 = 0$$

$$\Rightarrow \begin{aligned} x_2 &= -\frac{1}{2}y_2 \\ y_2 &= 2z_2 \end{aligned}$$

If  $z_2 = k_2$  then  $y_2 = 2k_2$  and  $x_2 = -k_2$

If  $k_2 = 1$   $x_2 = -1$   $y_2 = 2$   $z_2 = 1$

Hence Eigen vector corresponding to  $\lambda = -3$

$$x_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

(iii) Eigen Vector corresponding to  $\lambda = 5$

$$\begin{bmatrix} -4 & 2 & 0 \\ 2 & -4 & -6 \\ 2 & -2 & -2 \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Applying  $R_2 \rightarrow 2R_2 + R_1$

$R_3 \rightarrow 2R_3 + R_1$

$$\begin{bmatrix} -4 & 2 & 0 \\ 0 & -6 & -12 \\ 0 & -2 & -4 \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \rightarrow 3R_3 - R_2$

$$\begin{bmatrix} -4 & 2 & 0 \\ 0 & -6 & -12 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-4x_3 + 2y_3 = 0 \Rightarrow x_3 = \frac{1}{2}y_3$$

$$-6y_3 - 12z_3 = 0 \Rightarrow y_3 = -2z_3$$

$$x_3 = -z_3, \quad y_3 = -2z_3$$

If  $z_3 = 1$  then  $x_3 = -1$   $y_3 = -2$

Hence Eigen vector  $x_3 = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$



Q.3 State Cayley Hamilton theorem use it to find  $A^{-1}$

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Ans Statement:- Every Non Singular <sup>square</sup> Matrix satisfy its own characteristic equation.

Here. characteristic eq. of matrix A.

$$\begin{vmatrix} 0-\lambda & 1 & 2 \\ 1 & 2-\lambda & 3 \\ 3 & 1 & 1-\lambda \end{vmatrix} = 0$$

$$-\lambda [(2-\lambda)(1-\lambda)-3] - 1 [1(1-\lambda)-9] + 2 [1-3(2-\lambda)] = 0$$

$$-(\lambda^3 - 3\lambda^2 - 8\lambda + 2) = 0$$

$$\lambda^3 - 3\lambda^2 - 8\lambda + 2 = 0$$

As per Cayley. theorem.

$$A^3 - 3A^2 - 8A + 2I = 0$$

• Multiply by  $A^{-1}$

$$A^2 - 3A - 8I + 2A^{-1} = 0$$

$$-2A^{-1} = A^2 - 3A - 8I$$

$$A^{-1} = -\frac{1}{2} [A^2 - 3A - 8I] \quad \text{--- (i)}$$

$$A^2 = A \cdot A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 4 & 5 \\ 11 & 8 & 11 \\ 4 & 6 & 10 \end{bmatrix}$$

Put  $A^2$ , A, in I

$$A^{-1} = -\frac{1}{2} \left[ \begin{bmatrix} 7 & 4 & 5 \\ 11 & 8 & 11 \\ 4 & 6 & 10 \end{bmatrix} - 3 \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} - 8 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right]$$

$$= \frac{1}{2} \begin{bmatrix} 1 & -1 & 1 \\ -8 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix}$$



Q.4 Find the Fourier Series to represent

$$f(x) = x - x^2 \text{ in the interval } -1 < x < 1$$

Ans It is General interval problem with interval

$$2L = 2 \Rightarrow L = 1$$

the Fourier Series for this

$$f(x) = a_0 + \sum (a_n \cos n\pi x + b_n \sin n\pi x)$$

$$\begin{aligned} \text{where } a_0 &= \frac{1}{2} \int_{-1}^1 f(x) dx = \frac{1}{2} \int_{-1}^1 (x - x^2) dx = \frac{1}{2} \int_{-1}^1 x^2 dx \\ &= - \int_0^1 x^2 dx = - \left( \frac{x^3}{3} \right)_0^1 = -\frac{1}{3} \end{aligned}$$

$$a_n = \frac{1}{1} \int_{-1}^1 (x - x^2) \cos n\pi x dx = -2 \int_0^1 x^2 \cos n\pi x dx \quad \left[ \begin{array}{l} x \cos n\pi x \\ \text{is odd function} \end{array} \right]$$

$$= -2 \left[ -x^2 \frac{\sin n\pi x}{n\pi} + 2x \frac{\cos n\pi x}{n^2 \pi^2} - \frac{2 \sin n\pi x}{n^3 \pi^3} \right]_0^1$$

$$= -2 \left[ \frac{2 \cos n\pi}{n^2 \pi^2} \right] = \frac{-4(-1)^n}{n^2 \pi^2} \quad \left[ \begin{array}{l} \sin n\pi = 0 \\ \cos n\pi = (-1)^n \end{array} \right]$$

$$b_n = \frac{1}{1} \int_{-1}^1 (x - x^2) \sin n\pi x dx = 2 \int_0^1 x \sin n\pi x dx$$

$$= 2 \left[ \left( \frac{x \cos n\pi x}{-n\pi} \right)_0^1 - \int_0^1 \frac{\cos n\pi x}{-n\pi} dx \right]$$

$[x^2 \sin n\pi x \text{ is odd}]$

$$= 2 \left[ -\frac{\cos n\pi}{n\pi} + \frac{\sin n\pi}{n^2 \pi^2} \right] = \frac{-2(-1)^n}{n\pi}$$

Therefore the fourier series of  $f(x)$  is

$$\begin{aligned} f(x) = x - x^2 &= \frac{1}{3} + \frac{4}{\pi^2} \left( \frac{\cos \pi x}{1^2} - \frac{\cos 2\pi x}{2^2} + \frac{\cos 3\pi x}{3^2} - \dots \right) \\ &\quad + \frac{2}{\pi} \left( \frac{\sin \pi x}{1} - \frac{1}{2} \sin 2\pi x + \frac{1}{3} \sin 3\pi x - \dots \right) \end{aligned}$$



Q.5 Obtain the First Two Harmonic of Fourier Series.

$x$	0	1	2	3	4	5
$y$	4	8	15	7	6	2

Ans Here the length of interval  $N = 2L = 6 \Rightarrow L = 3$ .

$x$	$y$	$\theta = \pi x/3$	$\cos \theta$	$\sin \theta$	$\cos 2\theta$	$\sin 2\theta$
0	4	0	1	0	1	0
1	8	$\pi/3$	$1/2$	$\sqrt{3}/2$	$-1/2$	$\sqrt{3}/2$
2	15	$2\pi/3$	$-1/2$	$\sqrt{3}/2$	$-1/2$	$\sqrt{3}/2$
3	7	$\pi$	-1	0	1	0
4	6	$4\pi/3$	$-1/2$	$-\sqrt{3}/2$	$-1/2$	$-\sqrt{3}/2$
5	2	$5\pi/3$	$1/2$	$-\sqrt{3}/2$	$-1/2$	$-\sqrt{3}/2$

The First two Harmonic

$$f(x) = a_0 + \left( a_1 \cos \frac{\pi x}{3} + b_1 \sin \frac{\pi x}{3} \right) + \left( a_2 \cos \frac{2\pi x}{3} + b_2 \sin \frac{2\pi x}{3} \right)$$

$$a_0 = \frac{1}{2N} \sum y = \frac{1}{6} (42) = 7$$

$$a_1 = \frac{2}{N} \sum y \cos \theta = \frac{2}{6} \left[ 4 \times 1 + 8 \times \frac{1}{2} - 15 \times \frac{1}{2} - 7 \times (-1) - 6 \times \frac{1}{2} + 2 \times \frac{1}{2} \right]$$

$$= \frac{1}{3} \left[ -\frac{17}{2} \right] = -\frac{17}{6} = -2.8$$

$$a_2 = \frac{2}{N} \sum y \cos 2\theta = \frac{1}{3} \left( -\frac{19}{6} \right) = -1.5$$

$$b_1 = \frac{2}{N} \sum y \sin \theta = \frac{1}{3} \left[ 4 \times 0 + 8 \times \frac{\sqrt{3}}{2} + 15 \times \frac{\sqrt{3}}{2} + 7 \times 0 - 6 \times \frac{\sqrt{3}}{2} - 2 \times \frac{\sqrt{3}}{2} \right]$$

$$= \frac{1}{3} \left[ 4\sqrt{3} + 15\sqrt{3} - 3\sqrt{3} - \sqrt{3} \right] = \frac{5}{2}\sqrt{3}$$

$$b_2 = \frac{2}{N} \sum y \sin 2\theta = \frac{1}{3} \left[ 4 \times 0 + 4\sqrt{3} + 15 \times \frac{\sqrt{3}}{2} + 7 \times 0 + 3\sqrt{3} - \sqrt{3} \right]$$

$$= \frac{1}{3} \left[ 8\sqrt{3} + \frac{15\sqrt{3}}{2} \right] = \frac{27}{6}\sqrt{3} = \frac{9}{2}\sqrt{3}$$

$$f(x) = 7 + \left( -2.8 \cos \frac{\pi x}{3} + \frac{5}{2}\sqrt{3} \sin \frac{\pi x}{3} \right) + \left( -1.5 \cos \frac{2\pi x}{3} + \frac{9}{2}\sqrt{3} \sin \frac{2\pi x}{3} \right)$$

Ans